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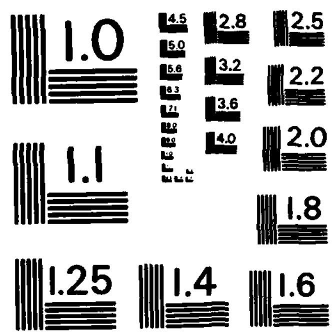

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STRENGTH OF MECHANICALLY FASTENED COMPOSITE JOINTS

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FOR THE COMMANDER

F. D. Cherry

F. D. CHERRY, Chief
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A method is presented for predicting the failure strength and failure mode of mechanically fastened joints made of fiber reinforced composite laminates. The method includes two steps. First, the stress distribution in the laminate is calculated by the use of a finite element method. Second, the failure load and the failure mode are predicted by means of a proposed failure hypothesis together with Yamada's failure criterion. A computer code was developed which can be used to calculate the maximum load and the mode of failure of joints involving laminates with different ply orientations,

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different material properties, and different geometries. Results generated by the present method were compared to data and to existing analytical and numerical solutions. The results of the present method were found to agree well with those reported previously. Parametric studies were also performed to evaluate the effects of joint geometry and ply orientation on the failure strength and on the failure mode.

FOREWORD

This report was prepared by Fu-kuo Chang, Richard A. Scott, and George S. Springer, Department of Mechanical Engineering and Applied Mechanics, The University of Michigan for the Mechanics and Surface Interactions Branch (AFWAL/MLBM), Nonmetallic Materials Division, Materials Laboratory, Air Force Wright Aeronautical Laboratories, Wright-Patterson AFB, Ohio. The work was performed under Contract Number F 33615-81-C-5050, Project number FY1457-81-02013.

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A	Total surface area of the laminate
A_L	Stress prescribed area
A_F	Stress free area
A_R	Displacement prescribed area (fixed boundary)
A_{Lg}	Surface area of an element g on which surface tractions is applied
B	Bearing stress
D	Diameter of the hole
E	Edge distance
E_{ijkl}	Elastic moduli
E_{mn}	Reduced laminate moduli
e	Failure indicator ($e<1$ non-failure, $e\geq 1$ failure)
e_o	Maximum value of e on the characteristic curve
$F_{i\beta}$	Assembled load vector
H	Thickness of the laminate
h^p	Thickness of the p-th ply
$k^g_{i\beta k\alpha}$	Stiffness matrix of the g-th element
$\bar{K}_{i\beta k\alpha}$	Assembled stiffness matrix
L	Plate length
M	Number of elements
N	Number of plies in the laminate
N_α	Shape function
n_j	Unit vector normal to the surface
P	Applied Load
P_{max}	Failure (maximum) load
$\bar{Q}^p_{ij}$	Transformed reduced stiffness matrix of the p-th ply

$q_{i\alpha}$	Nodal displacement
r	Radial distance
r_c	Radial distance to the characteristic curve
R_{ot}	Characteristic length for tension
R_{oc}	Characteristic length for compression
S	Ply shear strength
S_c	Shear strength of cross-ply laminate
s	Total area of the two-dimensional laminate
s_g	Area of element g (2-dimensions)
SCF	Stress concentration factor
T_i	Surface traction components
u_i	Displacement
$\overline{u}_i$	Arbitrary displacement functions
V_o	Total volume of the laminate
v_g	Volume of element g
W	Width of the plate
X	Ply tensile strength
x	Coordinate along the fiber direction in each ply
x_1	Coordinate perpendicular to the loading direction in the laminate plane
x_2	Coordinate opposed to the loading direction and perpendicular to the x_1-axis
x_3	Coordinate perpendicular to the x_1 and x_2 axes
y	Coordinate perpendicular to the fiber direction in each ply
Γ_L	Boundary curve of the hole on which the surface traction is applied
Γ_{Lg}	Boundary curve of the element g on which the surface traction is applied
$\Delta\theta$	Ply continuity

ε_{ij}	Strain components in the x_1-x_2 coordinate system
η	Angle measured counterclockwise from the x_1-axis
θ_f	Angle at which failure occurs
σ_{ij}	Stress components in the x_1-x_2 coordinate system
$\sigma_x, \sigma_y, \sigma_{xy}$	Stress components in the x-y coordinate system
ϕ	Maximum angular range of the ply orientation

SECTION I

INTRODUCTION

Among the major advantages of laminated composite structures over conventional metal structures are their comparatively high strength to weight and stiffness to weight ratios. As a result, fiber reinforced composite materials have been gaining wide application in aircraft and spacecraft construction. These applications require joining composites either to composites or to metals. Most commonly, joints are formed using mechanical fasteners. Therefore, suitable methods must be found to determine the failure strengths of mechanically fastened joints. A knowledge of the failure strength would help in selecting the appropriate size joint in a given application.

Owing to the significance of the problem, several investigators have developed analytical procedures for calculating the strength of bolted joints in composite materials. Among the recent studies are those of Waszczak and Cruse [1], Agarwal [2], and Garbo and Ogonowski [3]. As will be discussed in Section VII, the previous methods provide conservative results and underestimate the failure strength, often by as much as fifty percent.

The major objective of this investigation was, therefore, to develop a method which a) predicts the failure strength and failure mode of mechanically fastened composite joints with better accuracy than the existing analytical methods and b) can be used readily in the design of mechanically fastened composite joints. In the present method first the stress distribution around the hole is calculated by the use of a finite element method. Second, the failure load and the failure mode are predicted by means of a proposed new failure

hypothesis together with Yamada's [4] failure criterion. On the basis of this analysis a computer code was developed which can be applied to joints involving laminates with different ply orientations, different material properties, and different configurations, including different hole sizes, hole positions, and joint thicknesses. Because of the accuracy of the method and the flexibility of the computer code, the code can be applied to the analysis and the design of mechanically fastened composite joints.

SECTION II

PROBLEM STATEMENT

Consider a plate (length L, width W, thickness H) made of N fiber reinforced unidirectional plies. The ply orientation is arbitrary, but must be symmetric with respect to the $x_3=0$ plane (symmetric laminate, Figure 1). Perfect bonding between each ply is assumed.

A hole of diameter D is located along the centerline of the plate ($x_1=0$) at a distance E from one end of the plate. A rigid pin (diameter D), supported outside the laminate, is inserted into the hole (Figure 1). A uniform tensile load P is applied to the plate, as shown in Figure 1. The load is parallel to the plate (in-plane loading) and is symmetric with respect to the centerline. Hence the load cannot create bending moments about either the x_1, x_2 or x_3 axes. Moreover, for symmetric laminates, in plane and bending effects are uncoupled. It is desired to find

1) the stresses and strains in each ply,
2) the maximum (failure) load (P_{max}) that can be applied before the joint fails, and
3) the mode of failure.

Point 2 refers to the fact that, according to experimental evidence, mechanically fastened joints under tensile loads generally fail in three basic modes referred to as tension mode, shearout mode, and bearing mode. The type of damage resulting from each of these modes is illustrated in Figure 2. The objective, listed in point 3 above, is to determine which of these modes will most be responsible for the failure.

The calculation proceeds in three steps. For a given geometry and

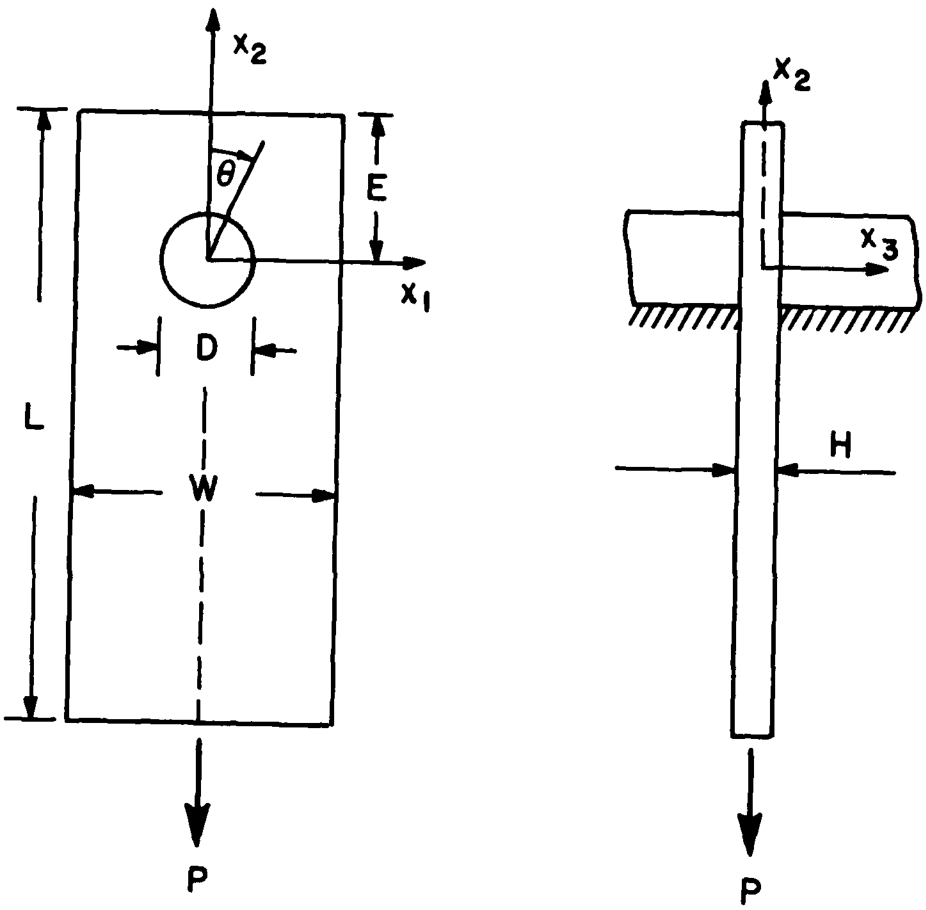

Figure 1. Geometry of the problem

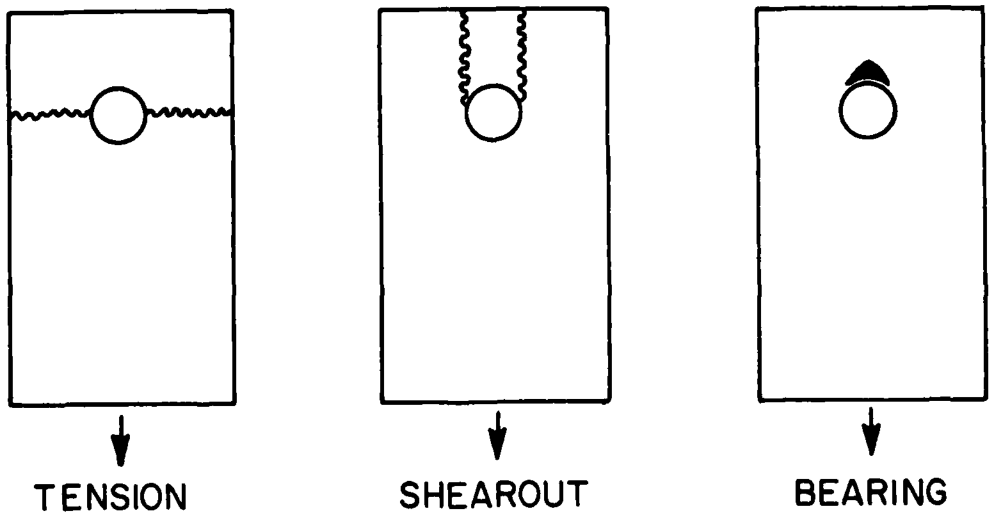

Figure 2. Illustration of the three basic failure modes

load

1) the stress distribution around the hole is calculated,

2) the maximum (failure) load is predicted , and

3) the mode of failure is determined.

The details of these steps are presented in Sections III-V.

SECTION III

STRESS ANALYSIS-GOVERNING EQUATIONS

The stresses in the laminate are calculated on the basis of anisotropic theory of elasticity and classical lamination plate theory. Accordingly, in the analysis planes are taken to remain planes, the strain across the thickness is taken to be constant [$\varepsilon_{ij}=f(x_1,x_2)$] and only plane stresses are considered ($\sigma_{13}=\sigma_{23}=\sigma_{33}=0$). Under these conditions, in the absence of body forces, the condition of force equilibrium can be expressed as [5]

$$\begin{aligned} \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} &= 0 \\ \frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} &= 0 \end{aligned} \tag{1}$$

In index notation eq. (1) becomes

$$\sigma_{ij,j} = 0 \tag{2}$$

σ_{ij} is the stress in the plane normal to the x_i axis and is in the x_j direction. The subscripts i and j may have the values 1 or 2. Consider now an elastic laminate of volume V_o containing a loaded hole as shown in Figure 3. Stresses are applied over the suface area A_L. The surface area A_R is rigidly fixed (no displacement), while the surface area A_F is free of applied stress. The total surface area is

$$A = A_L + A_R + A_F \tag{3}$$

Let us denote by $\bar{u}_i$ any arbitrary displacement inside the body. $\bar{u}_i$ is a test function. The only requirement is that $\bar{u}_i$ be continuous, differentiable and be zero on A_R. By multiplying eq. (2) by $\bar{u}_i$ and by taking the volume integral of the

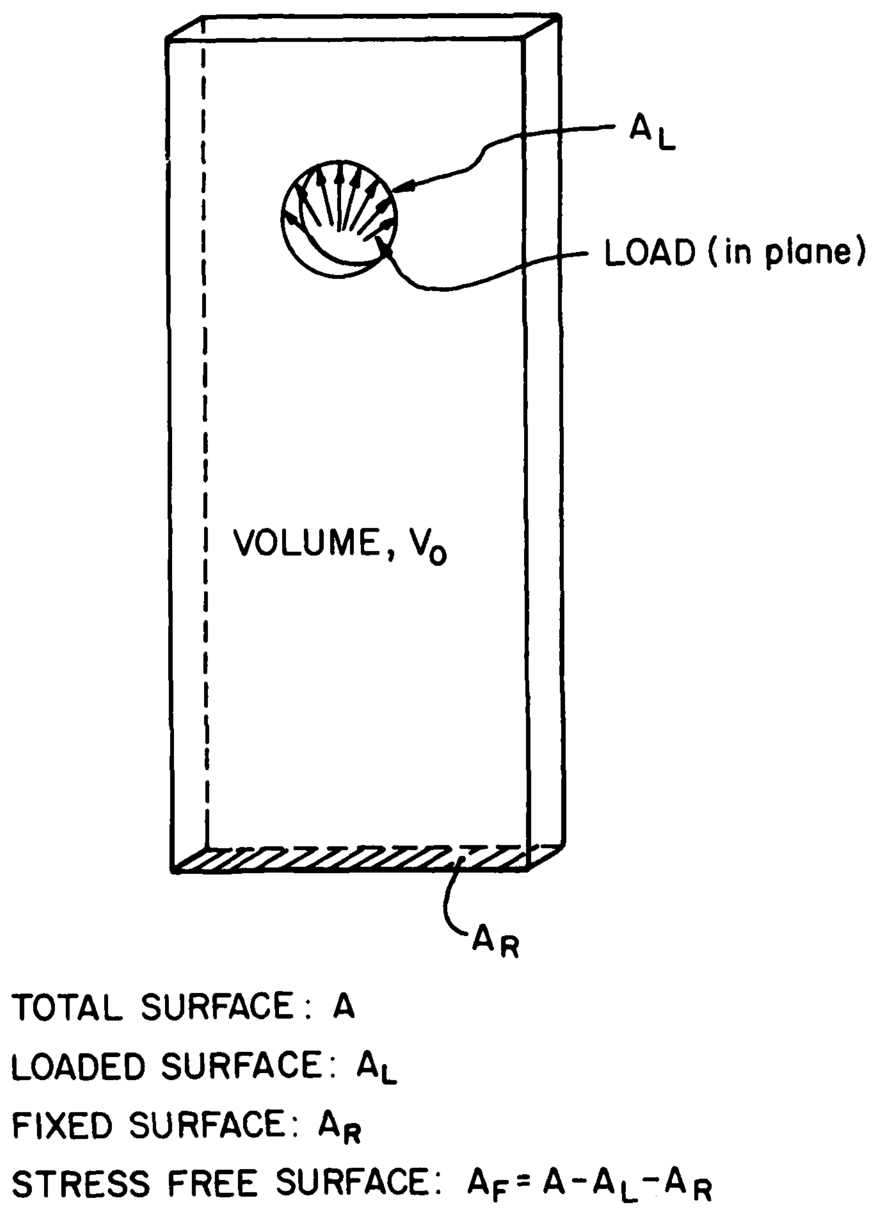

Figure 3. Configuration of an elastic laminate with a loaded hole

resulting expression we obtain

$$\iiint_{V_o} \sigma_{ij,j} \, \bar{u}_i \, dV = 0 \tag{4}$$

By employing the identity

$$\sigma_{ij,j} \, \bar{u}_i = (\sigma_{ij} \bar{u}_i)_{,j} - \sigma_{ij} \, \bar{u}_{i,j} \tag{5}$$

and by utilizing Gauss' theorem, eq. (4) may be written as

$$\iint_{A} \sigma_{ij} \, n_j \, \bar{u}_i \, dA - \iiint_{V_o} \sigma_{ij} \, \bar{u}_{i,j} \, dV = 0 \tag{6}$$

where n_j is the unit vector normal to the surface. On the free surface A_F the stresses are zero, while on the surface A_R the displacement is zero. These conditions give

$$\iint_{A_F} \sigma_{ij} \, n_j \, \bar{u}_i \, dA = 0 \tag{7}$$

$$\iint_{A_R} \sigma_{ij} \, n_j \, \bar{u}_i \, dA = 0 \tag{8}$$

The force per unit area (called surface traction) at each point of the surface area A_L is [5]

$$T_i = \sigma_{ij} n_j \tag{9}$$

Equations (6) - (9) yield

$$\iint_{A_L} T_i \, \bar{u}_i \, dA = \iiint_{V_o} \sigma_{ij} \, \bar{u}_{i,j} \, dV \tag{10}$$

The stress is related to the displacement through the stress-strain relationship, which for an elastic body is

$$\sigma_{ij} = E_{ijk\ell} \, \varepsilon_{k\ell} \tag{11}$$

The subscripts k and ℓ may take on the values of 1 or 2. The strains are related to the displacements u_i by the expression

$$\varepsilon_{k\ell} = \frac{1}{2}\left(\frac{\partial u_k}{\partial x_\ell} + \frac{\partial u_\ell}{\partial x_k}\right) \quad (12)$$

By combining eqs. (10) - (12) we obtain

$$\iiint_{V_o} E_{ijk\ell}\, \bar{u}_{i,j}\, u_{k,\ell}\, dV = \iint_{A_L} T_i\, \bar{u}_i\, dA \quad (13)$$

$E_{ijk\ell}$ are the moduli of elasticity of the laminate. Because of the laminate symmetry, the following simplification may be made

$$E_{ijk\ell} \equiv E_{mn} \quad (14)$$

The subscripts i,j,k, and ℓ are related to m and n as follows

$$\begin{array}{ll} i=j=1 \rightarrow m=1 & k=\ell=1 \rightarrow n=1 \\ i=j=2 \rightarrow m=2 & k=\ell=2 \rightarrow n=2 \\ i\neq j \rightarrow m=3 & k\neq\ell \rightarrow n=3 \end{array} \quad (15)$$

The reduced laminate modulus E_{mm} is given by

$$E_{mn} = \sum_{p=1}^{N} \frac{h^P}{H} \bar{Q}^P_{ij} \quad (16)$$

where h^P is the thickness of the p-th ply. $\bar{Q}^P_{ij}$ is the transformed reduced stiffness matrix for the p-th ply [6] (Appendix A).

SECTION IV

STRESS ANALYSIS-FINITE ELEMENT METHOD

In order to perform the calculations, the problem shown in Figure 1 was simulated by the geometry given in Figure 4. Because of symmetry the stresses were calculated only in one half of the body. Along the symmetry axis, displacement is allowed only in the x_2 direction. Along the lower edge of the plate, displacement is allowed only in the x_1 direction. The intersection of the symmetry axis and the lower edge is considered to be rigidly fixed.

The surface of the hole is subjected to a surface traction T_i. The parameter T_i is related to the applied load. The spatial distribution of T_i depends on the magnitude of the applied load, on the material properties, and on the geometry in a complex manner. It is extremely difficult, if not impossible, to determine the exact distribution of T_i inside the hole. To overcome this difficulty a cosine normal load distribution was assumed. With this approximation T_i becomes

$$T_i = -\frac{4P}{\pi D} n_i \cos\theta \tag{17}$$

The angle θ is in the x_1-x_2 plane and is measured clockwise from the x_2 axis (Figure 1). For isotropic materials the cosine normal load distribution (eq. 17) was found to represent closely the actual load distribution [7]. Calculations performed by previous investigators also showed that for composite materials the stress distribution inside the body is insensitive to the assumed load distribution [1,3,8]. Therefore, eq. (17) should suffice for the purpose of the present analysis which is to determine the overall strength of the joint.

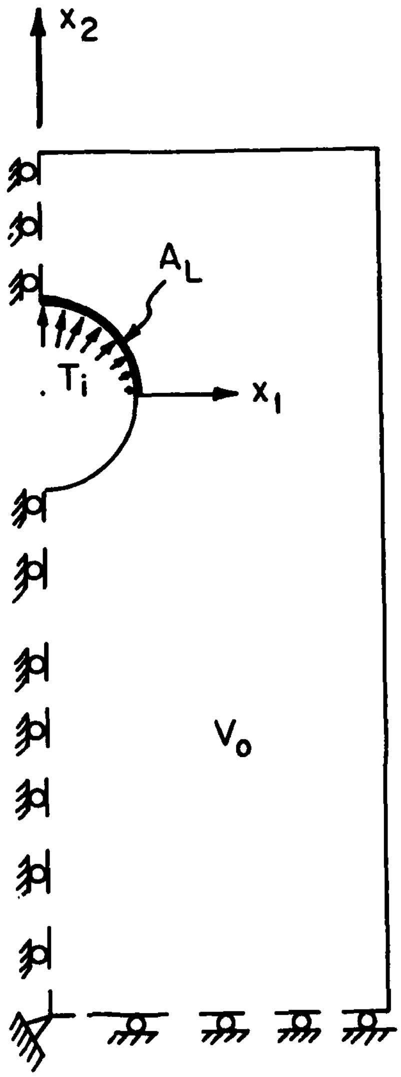

Figure 4. Configuration of a joint approximated in the finite element method

Equations (13) and (17) give

$$\iiint_{V_o} E_{ijk\ell}\, \bar{u}_{i,j}\, u_{k,\ell}\, dV = \iint_{A_L} -\frac{4P}{\pi D} n_i\, u_i \cos\theta\, dA \tag{18}$$

We recall that $\bar{u}_i$ are functions that can be selected arbitrarily. The unknowns in eq. (18) are the displacements $\bar{u}_i$. Once $\bar{u}_i$ are known the stress at every point can be calculated from eqs (11) and (12). The method of solution of eq. (18) is described below.

Solutions to eq.(18) were obtained by a finite element method (FEM). As a first step in the solution procedure the volume V_o is subdivided into M subdomains of volume V_g

$$V_o = \sum_{g=1}^{M} V_g \tag{19}$$

Equation (18) may now be written as

$$\sum_{g=1}^{M} \iiint_{V_g} E_{ijk\ell}\, \bar{u}_{i,j}\, u_{k,\ell}\, dV = \sum_{g=1}^{M} \iint_{A_{Lg}} -\frac{4P}{\pi D} n_i\, \bar{u}_i \cos\theta\, dA \tag{20}$$

A_{Lg} is the surface of an element on the inside of the hole where the surface traction is applied ($0 \leq \theta \leq \pi/2$). At any surface where load is not applied, A_{Lg} is zero. Advantage is taken now of the assumption that the strains ($\varepsilon_{11}, \varepsilon_{22}$ and ε_{12}) are independent of the thickness, i.e. the strains are independent of x_3 and depend only on x_1 and x_2. Thus, the three dimensional grid consisting of M volume elements may be replaced by a two dimensional grid consisting of M surface elements of area s_g (Figure 5)

$$s = \sum_{g=1}^{M} s_g \tag{21}$$

Equation (20) thus becomes

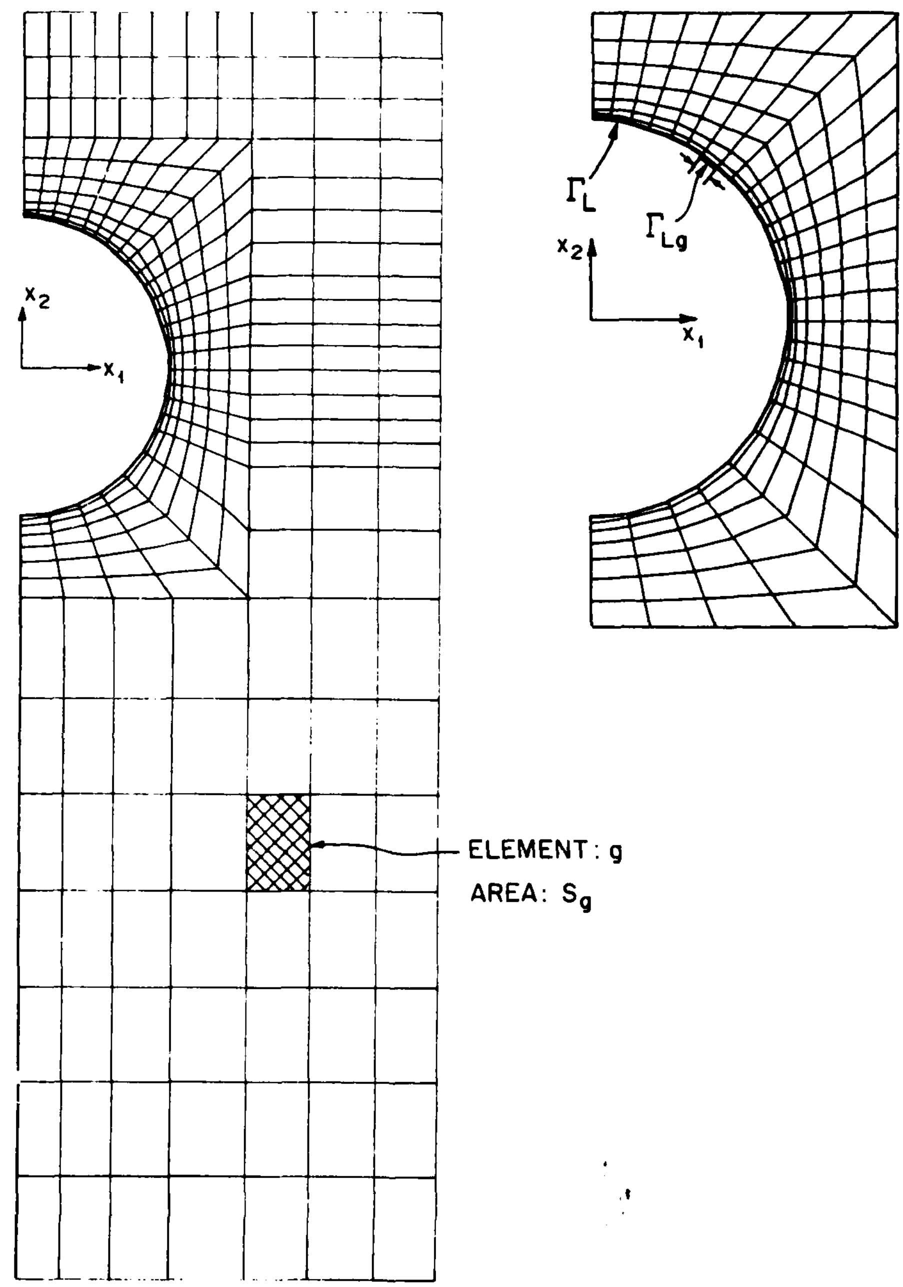

Figure 5. Grid used in the finite element method. Right hand figure is an enlarged view of the grid around the hole

$$\sum_{g=1}^{M} \iint_{S_g} E_{ijk\ell}\, \bar{u}_{i,j}\, u_{k,\ell}\, ds = \sum_{g=1}^{M} \int_{\Gamma_{Lg}} -\frac{4P}{\pi D}\, n_i\, \bar{u}_i \cos\theta\, d\Gamma \tag{22}$$

Γ_L is a line along the boundary of the hole where the surface traction is applied ($0 \le \theta \le \pi/2$, Figure 5). Each segment of this line (denoted by Γ_{Lg}) coincides with the boundary of an element g adjacent to the hole. For those elements which do not lie along this line Γ_{Lg} equals zero. Isoparametric 4-node elements were used in this investigation. The grid was generated using a grid generator [9]. The grid sizes were unequal. Smaller grids were used in the vicinity of the hole to obtain a better resolution of the stresses. Utilizing the symmetry about the x_2 axis, a grid (consisting of 306 elements) was placed on one half of the laminate, as illustrated in Figure 5.

The displacement in each element can be expressed in terms of the displacements of the four nodal points [10]

$$\begin{aligned} u_i &= N_\alpha\, q_{i\alpha} \\ \bar{u}_i &= N_\alpha\, \bar{q}_{i\alpha} \end{aligned} \tag{23}$$

The subscript α designates the nodal points (α=1,2,3, or 4). N_α is the shape function described in detail in Appendix B. $q_{i\alpha}$ is the displacement at the nodal point α in the i direction.

We define a stiffness matrix for the g-th element as

$$K^g_{i\beta k\alpha} \equiv \iint_{S_g} E_{ijk\ell}\, N_{\alpha,\ell}\, N_{\ell,j}\, ds \tag{24}$$

$K^g_{i\beta k\alpha}$ is an eight by eight matrix. The subscript β may take on the values 1,2,3, and 4. The nodal displacements $q_{k\alpha}$ and $\bar{q}_{i\beta}$ are independent of the surface and line integrations. Accordingly, eqs. (22) - (24) yield

$$\sum_{g=1}^{M} K^{g}_{i\beta k\alpha}\, q_{k\alpha}\, \overline{q}_{i\beta} \;=\; \sum_{g=1}^{M} \overline{q}_{i\beta} \int_{\Gamma_{Lg}} - \frac{4P}{\pi D}\, n_i\, N_\beta \cos\theta \, d\Gamma \tag{25}$$

The nodal displacements $\overline{q}_{i\beta}$ are arbitrary functions and hence eq. (25) can be written

$$\overline{K}_{i\beta k\alpha}\, q_{k\alpha} \;=\; \overline{F}_{i\beta} \tag{26}$$

where the global stiffness matrix, $\overline{K}_{i\beta k\alpha}$ and the load vector $\overline{F}_{i\beta}$ are given by

$$\overline{K}_{i\beta k\alpha} \;\equiv\; \sum_{g=1}^{M} K^{g}_{i\beta k\alpha} \tag{27}$$

$$\overline{F}_{i\beta} \;\equiv\; \sum_{g=1}^{M} \int_{\Gamma_{Lg}} - \frac{4P}{\pi D}\, n_i\, N_\beta \cos\theta \, d\Gamma \tag{28}$$

The elements of $\overline{K}_{i\beta k\alpha}$ and the components of the vector $\overline{F}_{i\beta}$ are known. Hence, $q_{k\alpha}$ can be obtained from eq. (26) using the Gaussian elimination method [12]. Once $q_{k\alpha}$ are known the displacements u_i are calculated from eq. (23). A computer code was developed for performing the calculations and for generating solutions (section VI).

SECTION V

PREDICTION OF FAILURE

In order to determine the load at which a joint fails and the mode of failure, the conditions for failure must be established. In this investigation the joint is taken to have failed when the combined stresses have exceeded a prescribed limit in any of the plies along an approximately chosen curve (denoted as the characteristic curve). The combined stress limit is evaluated using the failure criterion proposed by Yamada [4]. The coordinates of the characteristic curve are established by extending Whitney and Nuismer's failure hypothesis [13] (developed for open, unloaded holes) to loaded holes.

1) Failure Criterion

Numerous criteria for failure have been proposed in the past [14,15,16,17]. Although the concepts underlying the different failure criteria may be different, the results of the various criteria are generally quite similar. In this investigation Yamada's failure criterion was adopted [4]. This criterion is based on the assumption that just prior to failure of the laminate every ply has failed due to cracks along the fibers. The validity of this assumption is supported by tests performed during this investigation with 64 ply graphite-epoxy (AS/3501-6) laminates. Yamada's criterion states that failure occurs when the following condition is met in any one of the plies

$$\left(\frac{\sigma_x}{X}\right)^2 + \left(\frac{\sigma_{xy}}{S_c}\right)^2 = e^2 \begin{cases} e < 1 & \text{no failure} \\ e \geqq 1 & \text{failure} \end{cases} \qquad (29)$$

σ_x and σ_{xy} are the longitudinal and shear stresses in a ply, respectively (x and y being the coordinates parallel and normal to the fibers in the ply). X is the longitudinal tensile strength of the ply. S_c is the shear strength of a symmetric, cross ply laminate which has the same number of plies as the laminate under consideration. As indicated in eq. (29) failure occurs when e is equal to or greater than unity.

2) Failure Hypothesis-Characteristic Curve

The hypothesis is proposed here that failure occurs when in any one of the plies the combined stresses satisfy an appropriately chosen failure criterion at any point on a characteristic curve. The characteristic curve (Figure 6) is specified by the expression

$$r_c(\theta) = D/2 + R_{ot} + (R_{oc}-R_{ot}) \cos \theta \qquad (30)$$

The angle θ, measured clockwise from the x_2 axis, may range in value from $-\pi/2$ to $\pi/2$. R_{ot} and R_{oc} are the characteristic lengths for tension and compression [13,18]. These parameters can be determined experimentally by measuring the tensile and compressive strengths of notched laminates. R_{ot} and R_{oc} depend only on the material. Therefore, the coordinates of the characteristic curve also depend only on the material, and are independent of the geometry and the stress distribution.

In this investigation the characteristic curve is used together with the Yamada failure criterion. Accordingly (see eq. 29), failure occurs when the parameter e is equal to or is greater than unity at any point on the characteristic curve

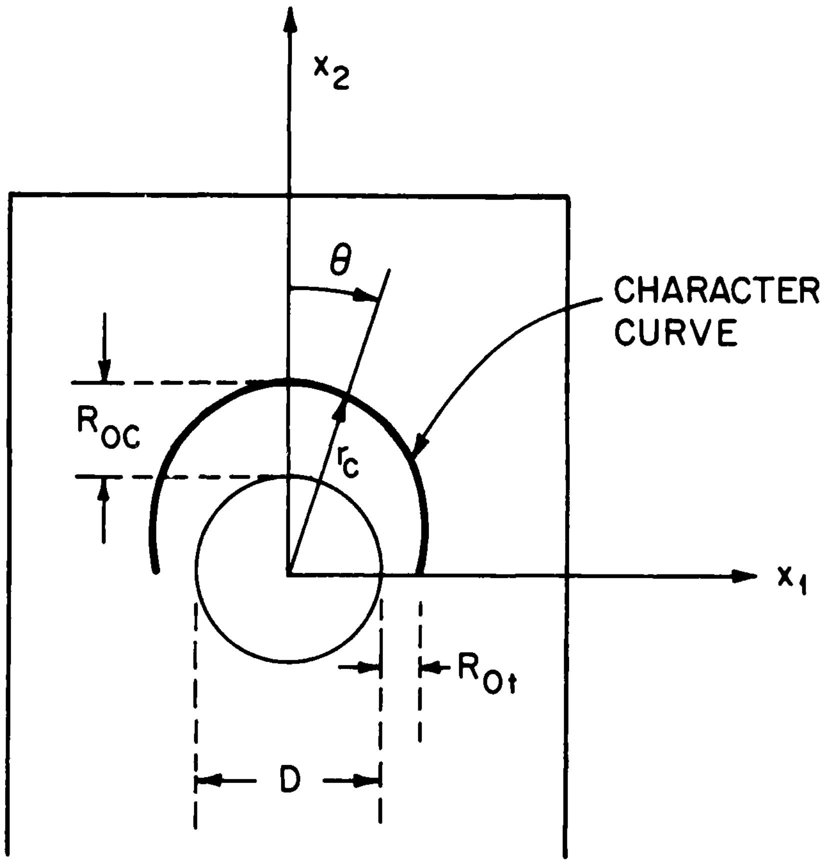

Figure 6. Description of the characteristic curve

$$
\left.\begin{array}{ll} \text{No failure} & e < 1 \\ \text{Failure} & e \geq 1 \end{array}\right\} \text{ at } r = r_c \tag{31}
$$

It is emphasized that the above failure hypothesis is used here in conjunction with the Yamada failure criterion (eq. 29). However, the hypothesis is general, and is not restricted to Yamada's criterion. The characteristic curve proposed here may be used with any other failure criterion.

3) Solution Procedure

Whether or not a joint fails under a given condition is determined as follows. For a given load

a) the stresses ($\sigma_1, \sigma_2, \sigma_{12}$) are calculated in each ply using eqs. (11), (14) and (16), and the FEM described in Sections II, and III,

b) the longitudinal and shear stresses (σ_x, σ_{xy}) are evaluated in each ply employing the transformation

$$
\begin{aligned}
\sigma_x &= \sigma_1 \cos^2\eta + \sigma_2 \sin^2\eta + 2\sigma_{12} \sin\eta \cos\eta \\
\sigma_{xy} &= -\sigma_1 \sin\eta \cos\eta + \sigma_2 \sin\eta \cos\eta + \sigma_{12}(\cos^2\eta - \sin^2\eta)
\end{aligned} \tag{32}
$$

where η is the angle measured counter clockwise from the x_1-axis to the x-axis of each ply.

c) the parameter e is calculated (eq. 29) along the characteristic curve

d) if e equals or exceeds the value of unity ($e \geq 1$) in any ply along the characteristic curve, the joint is taken to have failed.

The procedure outlined above is used to predict whether or not failure occurs under a given load. Due to the assumption of a cosine normal load distribution around the hole (eq. 17), the calculated

stresses are linearly proportional to the applied load P. This fact together with Yamada's failure criterion (eq. 29) gives

$P \sim e$

This relationship is utilized to determine the maximum load (P_{max}) which can be imposed on the joint. For a given load P, values of e are calculated on the characteristic curve as discussed above (points a-d). The highest value of e (e_o) is then determined, and the maximum load is calculated by the expression

$$P_{max} = \frac{P}{e_o} \tag{34}$$

The calculation procedure described in the foregoing also provides the location (angle θ_f) at which e first reaches the value of unity (e=1) on the characteristic curve. (Figure 7). A knowledge of θ_f provides an estimate of the mode of failure. When θ_f is small ($\theta_f \approx 0°$) failure is by the bearing mode. When $\theta_f \approx 45°$ failure is due to shearout. When $\theta_f \approx 90°$ failure is caused by tension. In summary

$$\begin{array}{ll} -15° < \theta_f < 15° & \text{bearing mode} \\ 30° < \theta_f < 60° & \text{shearout mode} \\ 75° < \theta_f < 90° & \text{tension mode} \end{array} \tag{35}$$

At intermediate values of θ_f failure may be caused by a combination of these modes.

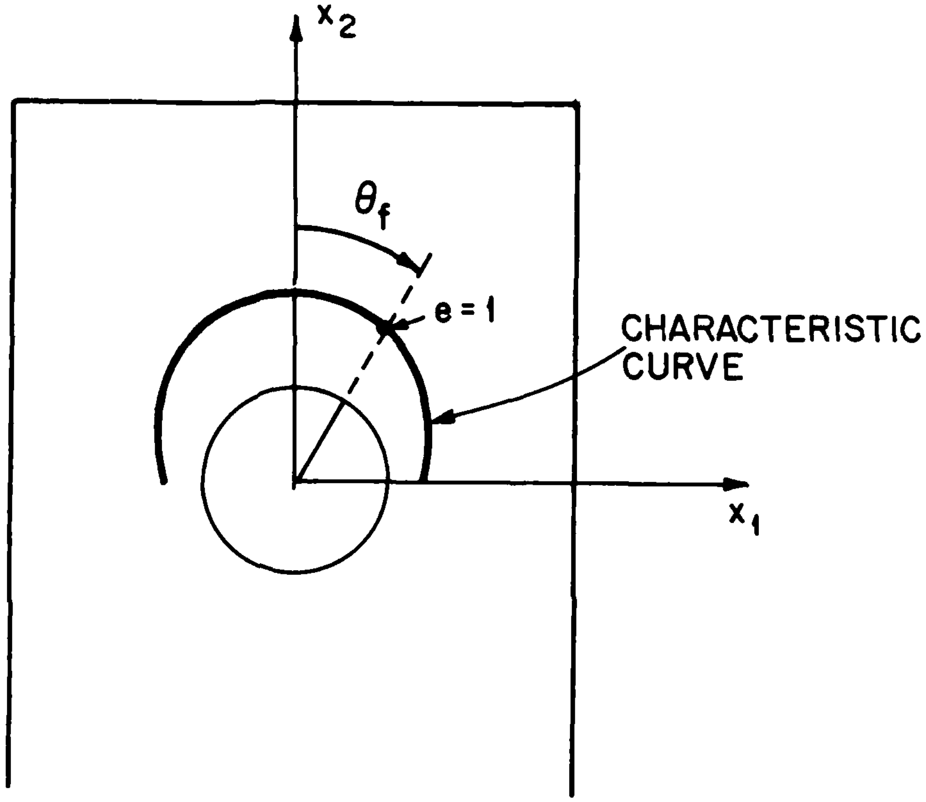

Figure 7. Location of failure (e=1) along the characteristic curve

SECTION VI

NUMERICAL SOLUTION

A computer code (designated as BOLT) was developed which is suitabe for generating solutions to the problem formulated in Sections II-V. The required input parameters and the output provided by the code are summarized in Table 1.

A Fortran listing of the code and a sample input-output are included in Appendix C.

Table 1. Input parameters required by the computer code and the output provided by the code.

Input Parameters

1) Ply properties

   a) Young's modulus, E_1

   b) Shear modulus, G_{12}

   c) Poisson's ratio, ν_{12}

2) Ply orientations

3) Geometry

   a) hole diameter, D

   b) thickness, H

   c) width, W

   d) length, L

   e) edge distances, E

4) Characteristic lengths, R_{ot} and R_{oc}

5) Longitudinal tensile strength of each ply, X

6) Shear strength of cross ply laminate S_c

Output Parameters

1) Failure load

2) The failure mode

3) Stresses in the laminate

SECTION VII

RESULTS AND DISCUSSIONS

Results were generated in order to assess the validity and accuracy of the method and the computer code and to compare the results of the present method with the results of other existing methods of solutions. In addition, parametric studies were performed to evaluate the major characteristics of bolted joints.

1) Isotropic and Orthotropic Plates

Stress distributions were calculated in isotropic plates containing both unloaded (open) and loaded holes and in orthotropic plates containing unloaded holes. These problems were selected because analytical solutions are available for comparisons with the results of the present method.

An analytical solution for the stress distribution in an infinite ($W \to \infty$) isotropic plate containing an unloaded hole was given by Timoshenko [19]. The stress distribution in such a plate was also calculated by the present method. The parameters used in the numerical calculations are given in Figure 9. A large width (W/D=14) was used in the calculation to approximate an infinite plate. The results of the present method and the analytical solution of Timoshenko are compared in Figure 8. There is excellent agreement between the stresses calculated by the two methods.

The stresses in isotropic plates containing loaded holes were also calculated. Plates of infinite and finite widths were considered. Calculations were performed for the parameters given in Figure 9 and Table 2. From the calculated stresses, the stress concentration factor was determined. The stress concentration factor is defined as

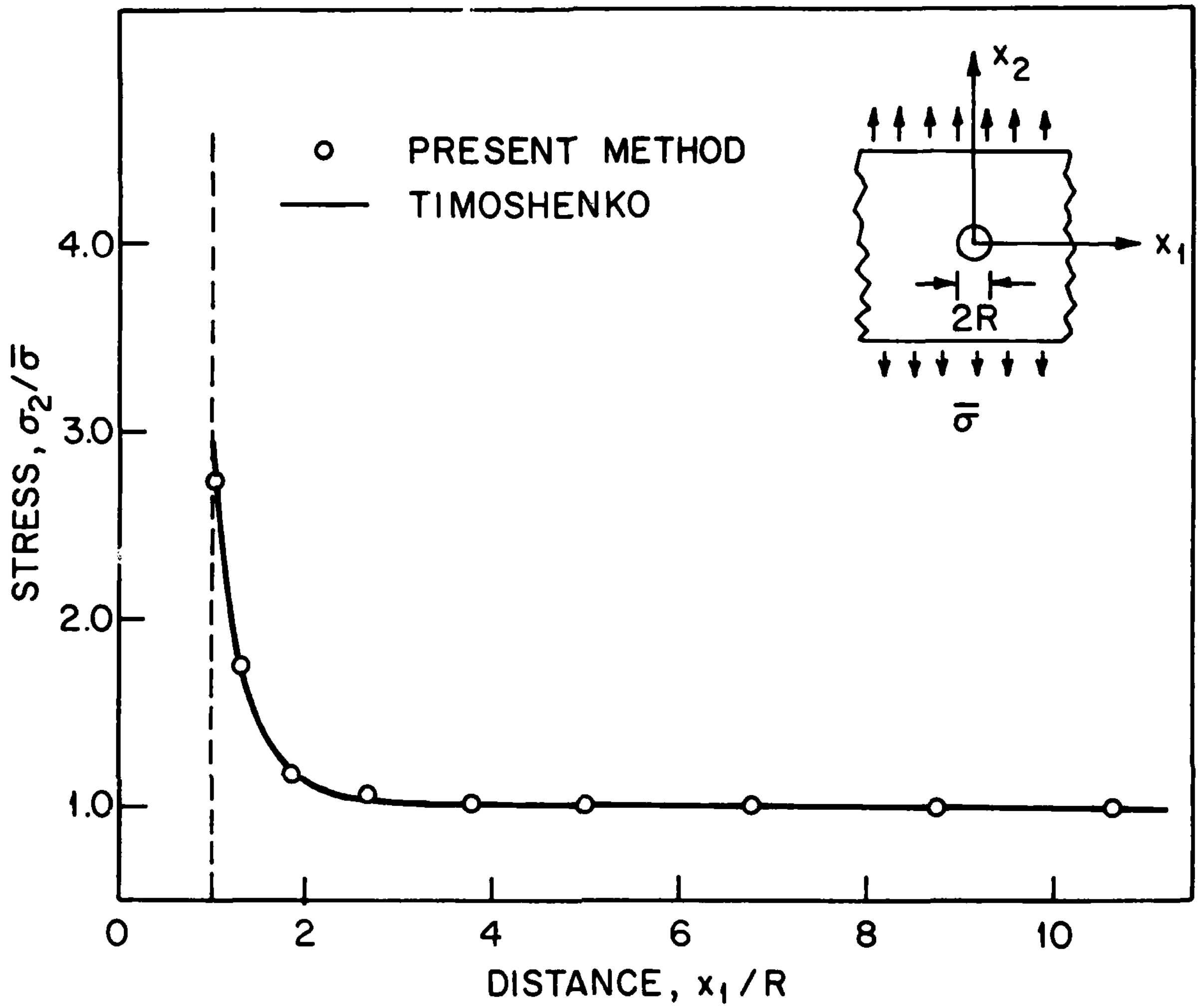

Figure 8. The stress σ_2 along the x_1-axis in an isotropic infinite plate containing a c'rcular hole. Comparison of the present results with the theoretical results given by Timoshenko [19]. Parameters used in the numerical calculations: $\bar{\sigma}$=1.64 MPa, D=2R=7.62 mm, W/D=14, E/D=14, L/D=28

$$SCF \equiv \frac{(\sigma_2)_{max}}{B} \quad (36)$$

where $(\sigma_2)_{max}$ is the maximum stress in the plate (perpendicular to the x_1-x_3 plane, Figure 1) and B is the bearing stress

$$B \equiv \frac{P}{(H)(D)} \quad (37)$$

The stress concentration factors obtained by the present method were compared to those reported by previous investigators (Table 2). The maximum difference in the stress concentration factors given by the different methods is about 20 percent. The stress concentration factor given by the present method differs from the values given by previous investigators at most by 15 percent.

The stresses in an isotropic plate of finite width containing a loaded hole are shown in Figure 9. The stresses calculated by the present method are in excellent agreement with De Jong's approximate solution [8].

The stress distribution in an orthotropic plate of finite width containing an open (unloaded) hole was also calculated. The calculations were performed for a plate with the symmetric laminate lay up of $[0/90]_s$. An analytical solution for this problem was provided previously by Nuismer and Whitney [18], who modified Lekhnitskii's earlier solution [20] for an infinite plate. The results given in Figure 10 show excellent agreement between the stresses calculated by the present method and by the analytical solution.

The aforementioned comparisons indicate that the present method predicts the stress distribution around loaded and unloaded holes with high accuracy.

Table 2. Stress Concentration Factor (SCF) Around a Pin Loaded Hole Contained in an Isotropic Plate of Infinite Width. Comparison of Present Result with Those Obtained by Previous Investigators.

Investigators	SCF
Present Result*	0.985
Hong [22]	0.955
De Jong [8]	1.058
Eshwar et al [23]	0.922
Bickley [7]	0.81

*Present results was calculated for the case:

$D = 7.62$ mm, $W/D = 8$, $E/D = 4$, $L/D = 14$.

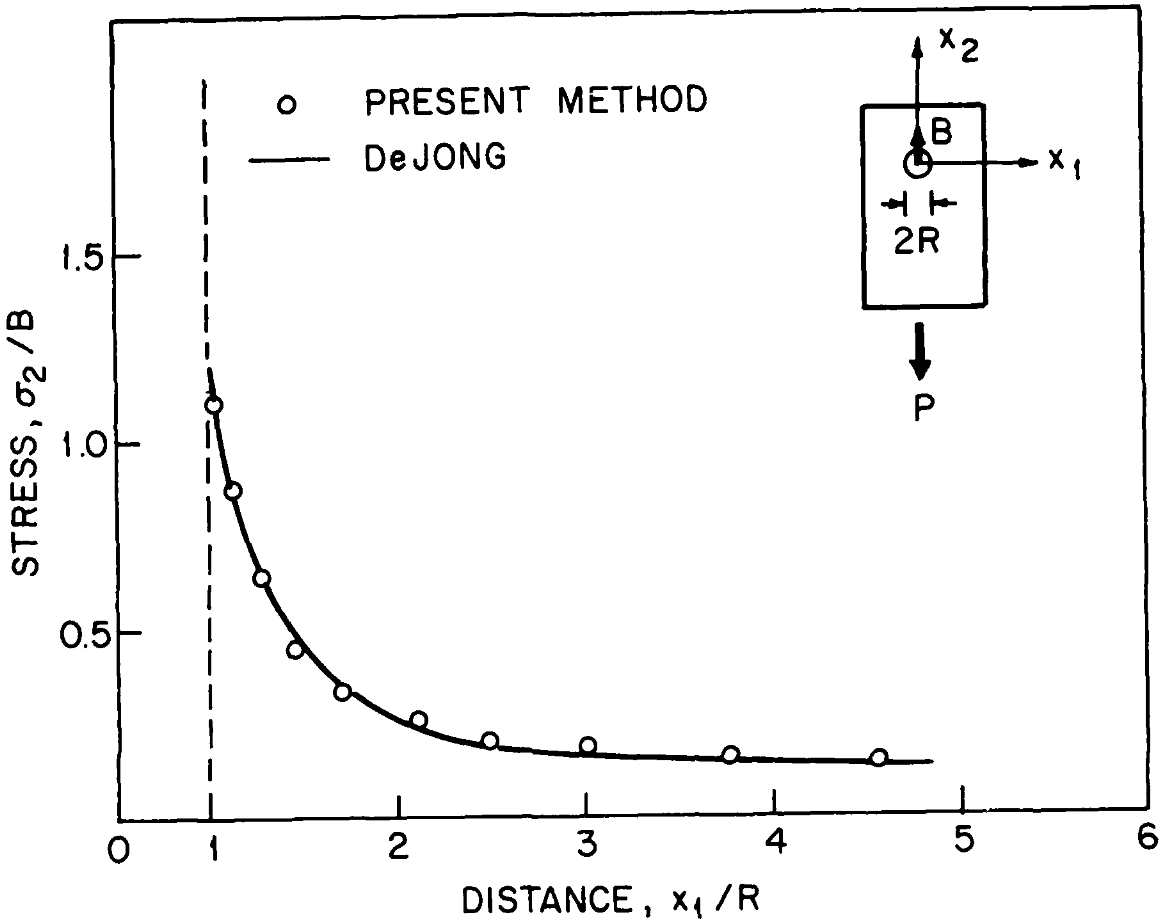

Figure 9. The stress σ_2 along the x_1-axis in an isotropic plate of finite width containing a loaded hole. Comparison of the present results with the theoretical results given by De Jong [8]. Parameters used in the numerical calculations: D=7.62 mm, W/D=5.0, E/D=4.0, L/D=14.0

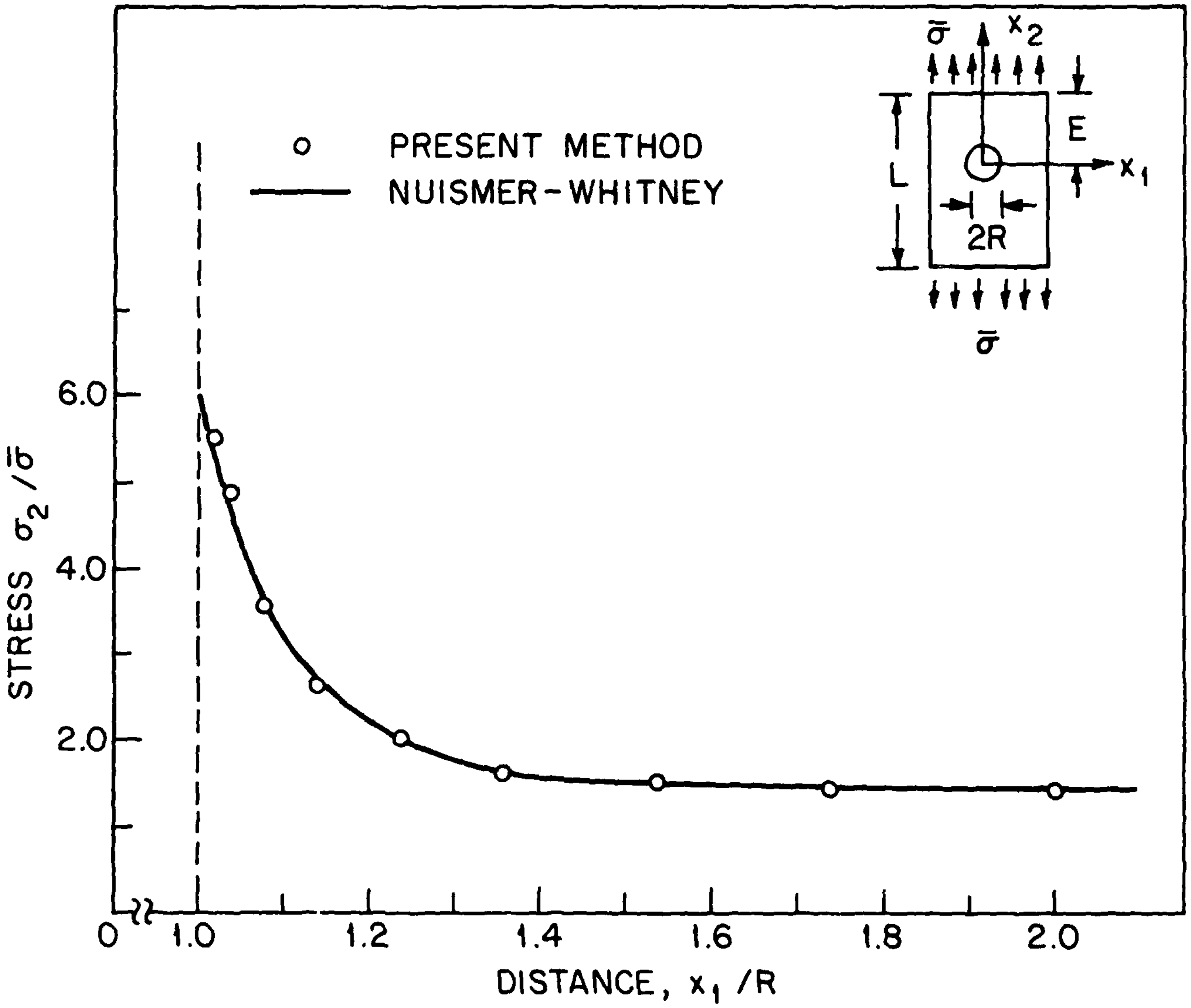

Figure 10. The stress σ_2 along the x_1-axis in an orthotropic finite plate $[0/90]_s$ containing a circular hole. Comparison of the present results with the theoretical results obtained by Nuismer and Whitney [18]. Parameters used in the numerical calculations: Material: Graphite/Epoxy T300/5208, E_1=149.8 GPa, E_2=11.2 GPa, G_{12}=5.39 GPa, ν_{12}=0.29, $\bar{\sigma}$=2.3 MPa, D=24.5 mm, W/D=3.0, E/D=4.0, L/D=14.0

2) Failure Strength and Failure Mode

Failure strengths of mechanically fastened composite joints calculated by the present method were compared to available data and to failure strengths predicted by other numerical methods.

Failure strengths of joints in graphite-epoxy laminates were measured by Van Siclen [21] and by Garbo and Oganowski [3]. The test conditions are summarized in Table 3. The failure strengths of the joints under these conditions were also calculated by the present method. The material properties used in the calculations are summarized in Table 4. Comparisons between the experimental and predicted failure strengths are given in Table 5. In these tables comparisons between the data and the failure strengths predicted by Agarwal [2] and by Garbo and Oganowski [3] are also included.

As can be seen, of the three analytical methods included in Table 5 the present one predicts the failure most accurately. In most cases, the failure load given by the present method agrees with the data within about 10%. On the other hand, the failure loads given by the Agarwal and by the Garbo and Oganowski methods are in error by as much as 50 percent.

One point is of interest here. When there is a high fraction of zero degree plies in the laminate (parallel to the body direction) the laminate fails in the shearout mode. In this case the calculated results are sensitive to the value of the laminate shear strength S_c. For example, when the S_c value obtained with cross ply laminates is used to calculate the failure load of $[0/\pm45/90]_s$ laminates containing 70 percent of 0° plies the calculated and

Table 3. Summary of Test Conditions

Investigator	Material	Case No.	D(mm)	W/D	E/D	L/D	H(mm)	Lay up	Volume Fraction %0°	%45°	%90°
Van Siclen [21]	T300/sp286	1	4.76	8.025	2.983	14.68	1.067	$[0/\pm45/90]_s$	25	50	25
		2	4.76	8.025	2.083	14.68	1.067	$[0/\pm45/90]_s$	25	50	25
		3	4.76	5.336	2.083	14.68	2.235	$[0/\pm45/90]_s$	25	50	25
		4	4.76	5.336	2.983	14.68	1.397	$[0_2/\pm45/90]_s$	40	40	20
		5	4.76	5.336	2.983	14.68	1.067	$[0/\pm45/90]_s$	25	50	25
		6	4.76	8.025	4.013	14.68	1.067	$[0/\pm45/90]_s$	25	50	25
		7	4.76	5.336	2.983	14.68	1.118	$[\pm45]_{2s}$	0	100	0
		8	4.76	5.336	2.983	14.68	1.118	$[0/90]_{2s}$	50	0	50
		9	4.76	5.336	2.983	14.68	1.067	$[0_2/\pm45]_s$	50	50	0
Garbo and Ogonowski [3]	AS/3501-6	10	6.35	6	3	14.68	5.283	$[0/\pm45/90]_s$	30	60	10
		11	6.35	6	3	14.68	5.283	$[0/\pm45/90]_s$	50	40	10
		12	6.35	6	3	14.68	5.283	$[0/\pm45/90]_s$	70	20	10

Table 4. Material Properties Used in the Calculations

Ply Properties	T300/SP286	AS/3501-6
Longitudinal Modulus E_1 Gpa(10 ksi)	130 (18.7)[21]	130(18.85)[3]
Transverse Modulus E_2 Gpa(10 ksi)	8.274(1.2)[21]	13.1(1.9)[3]
Shear Modulus G_{12} Gpa(10 ksi)	5.033(0.73)[21]	5.86(0.85)[3]
Poisson Ratio ν_{12}	0.30[21]	0.30[3]
Tensile Strength X Gpa(10 ksi)	1.23(0.178)[21]	1.58(0.23)[3]
Shear Strength S Gpa(10 ksi)	0.05(0.0073)[21]	0.12(0.017)[3]

Laminate Properties	T300/SP286	AS/3501-6
Cross-ply Laminate Shear Strength S_c Gpa(10 ksi)	0.125 [a](0.018)	0.204 [b](0.03) 0.12[c](0.017)
Characteristic Length R_{ot} mm(in)	1.092 [d](0.043)	0.584 (0.023)[3]
Characteristic Length R_{oc} mm(in)	3.048 [e](0.12)	1.727[e](0.068)

(a) Tests with Glass/Epoxy showed S_c to be about 2.5 times higher than the ply shear strength S [4]. The 2.5 multiplier was used to obtain S_c of Graphite/Epoxy T300/SP286 from the ply shear strength S given in [21].

(b) The value of S_c for AS/3501-6 was taken to be 1.7 times the ply shear strength S.

(c) This value was used for laminates containing more than 70% (by volume) of 0 degree plies.

(d) For T300/SP286 the value was chosen from [18] for T300/5208.

(e) The values of R_{oc} for T300/SP286 and AS/3501-6 were evaluated by the method given in [18] together with the data in [24,25].

Table 5. Comparisons Between the Experimental (P) and Calculated (P_c) Failure Loads. Case Numbers Correspond to Test Conditions Given in Table 3.

Material	Case No.	Lay Up	Percent Difference$(1-P/P_c)$x100 Present Results	Agarwal(1980)
T300/SP286	1	$[0/\pm45/90]_s$	3.7	8.7
	2	$[0/\pm45/90]_s$	0.01	0.01
	3	$[0/\pm45/90]_{2s}$	1.81	7.78
	4	$[0_2/\pm45/90]_s$	0.01	0.01
	5	$[0/\pm45/90]_s$	8.66	1.16
	6	$[0/\pm45/90]_s$	11.18	1.2
	7	$[\pm45]_{2s}$	11.1	49.43
	8	$[0/90]_{2s}$	12.3	49.85
	9	$[0_2/\pm45]_s$	7.64	20.0
			Present Results	Garbo and Ogonowski(1981)
AS/3501-6	10	$[0/\pm45/90]_s$	0.5	45
	11	$[0/\pm45/90]_s$	3.8	27
	12	$[0/\pm45/90]_s$	15.0	14

measured values differ by 35 percent. On the other hand, if the shear stress of the individual ply is used in the calculations the difference between the calculated failure load and the data is only 15 percent. This indicates that the value of S_c must be selected carefully.

Failure loads calculated by Waszczak and Cruse [1] are compared to data in Table 6. Waszczak and Cruse's method also yields failure loads which are in error by as much as 50 percent. The present method was not applied to Waszczak and Cruse's data because the material properties needed for the calculation were unavailable.

It is interesting to note that the accuracies of all four methods (present, Agarwal, Garbo and Ogonowski, and Waszczak and Cruse) depend on the arrangements of the plies in the laminate. In general, the the analytical predictions are most accurate for quasi-isotropic laminates, and are least accurate for angle ply and cross ply laminates. However, even for angle ply and cross ply laminates the present method yields results within about 10 percent accuracy, in contrast with the results of other existing methods of solutions, which may be in error by as much as 50 percent.

The failure modes predicted by the present method were also compared to failure modes observed experimentally. These comparisons, given in Table 7, show that the present method predicts well the mode of failure.

The aforementioned comparisons between the results of the present method and the data show that the method predicts with good accuracy both the load at which the joint fails and the mode of failure. The good agreements between the predictions of the model and the data

Table 6. Comparisons Between the Experimental Failure Loads and the Values Predicted by Waszczak and Cruse [1]

Material	Lay Up	Volume Fraction % 45	0	90	W/D	E/D	Percent Difference $(1-[P/P_c)x100$
Graphite/Epoxy	$[\pm 45]$	100	0	0	4	3	53
Boron/Epoxy	$[(\pm 45/\bar{0})_s/\overline{90}]_s$	72.3	18.2	9.1	4	2	24
	$[\pm 45_5/90_6]$	62.5	0	37.5	4	3	7
	$[\pm 45/(0_6/90)_s]_s$	13.3	80.0	6.7	6	6	42
	$[0_6/\pm 45_5]$	62.5	37.5	0	7.5	2.5	2
	$[0_6/\pm 45]$	62.5	37.5	0	7.35	4	35

Table 7. Comparisons of Predicted Failure Modes with those Observed Experimentally
T-Tension Mode S-Shearout Mode B-Bearing Mode

Material	Case No.	Lay Up	Observed Failure Mode	Predicted Failure Mode Present	Predicted Failure Mode Agarwal (1980)
T300/SP286	1	$[0/\pm45/90]_s$	S/B	S	S
	2	$[0/\pm45/90]_s$	S	S	S
	3	$[0/\pm45/90]_{2s}$	T/S	T	T
	4	$[0_2/\pm45/90]_s$	S	S	S
	5	$[0/\pm45/90]_s$	T	T	T
	6	$[0/ 45/90]_s$	B	B	B
	8	$[0/90]_{2s}$	S	S	S
	9	$[0_2/\pm45]_s$	B/S	S	S
AS/3501-6				Present	Garbo & Ogonowski (1981)
	10	$[0/\pm45/90]_s$	B/S	B	-
	11	$[0/\pm45/90]_s$	B/S	S	-
	12	$[0/\pm45/90]_s$	B/S	S	-

(illustrated in Tables 5-7) create further confidence in the model.

3) Effects of Geometry and Ply Orientations

Parametric studies were performed to evaluate the effects of joint geometry and ply orientation on the failure strength and on the failure mode.

The effects of joint width on joint failure is illustrated in Figure 11. In this, and in subsequent figures, the failure load is normalized with respect to the ultimate tensile load of the laminate (without the hole) in the direction of the applied load. As is shown by the results in Figure 11, in general, the maximum load the joint can carry decreases as the hole size decreases, when the width to hole diameter ratio is greater than about 3. As the hole diameter approaches the width ($W/D \rightarrow 1$) the strength reduces to zero ($P \rightarrow 0$). Here failure loads were not calculated for W/D less then three because at such low W/D ratios the assumption of the cosine load distribution (eq. 17) is inaccurate [8].

The effect of edge distance E (Figure 1) on the failure load is shown in Figure 12. For the lay-ups $[0/\pm 45/90]_s$ and $[0/90]_{2s}$ increasing edge distance results in higher failure loads, as long as E/D is less than about 4. For higher edge ratios ($E/D>4$) an increase in edge distance does not seem to influence significantly the failure load. For the lay-up $[0_2/\pm 45]_s$, the failure load does not vary significantly with E/D.

The effects of ply orientation on the failure load are given in Figure 13. This figure illustrates the effects of two parameters 1) the maximum ply angle ϕ in the laminate and 2) the change in

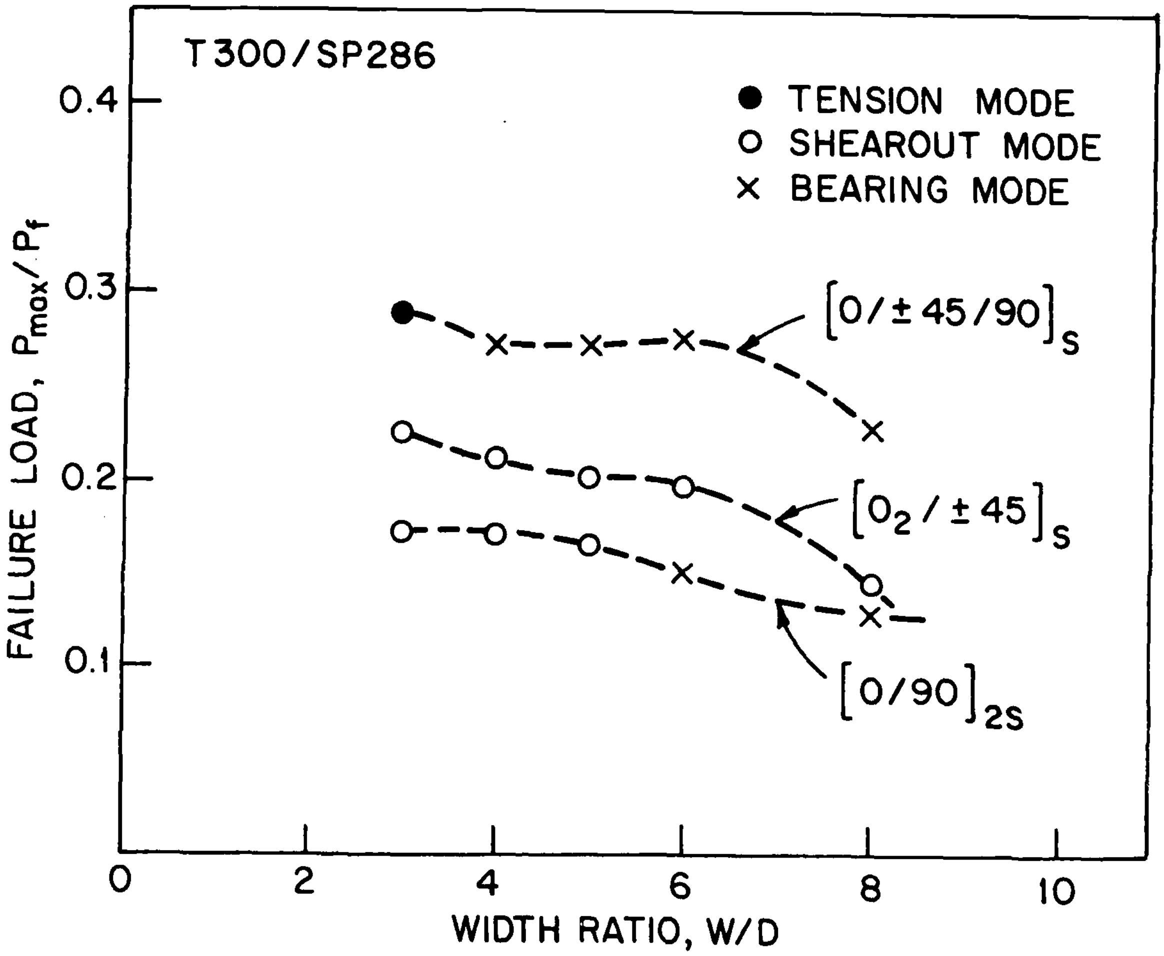

Figure 11. The effects of width ratio on the failure load of laminates with different ply orientations. P_f is the tensile failure load of laminates without holes. Parameters used in the numerical calculations: Material: Graphite/Epoxy T300/SP286, W=38 mm, E=50.8 mm, L=203.2 mm, H=1.067 mm for $[0/\pm 45/90]_s$ and $[0_2/\pm 45]_s$, and H=1.18 mm for $[0/90]_{2s}$

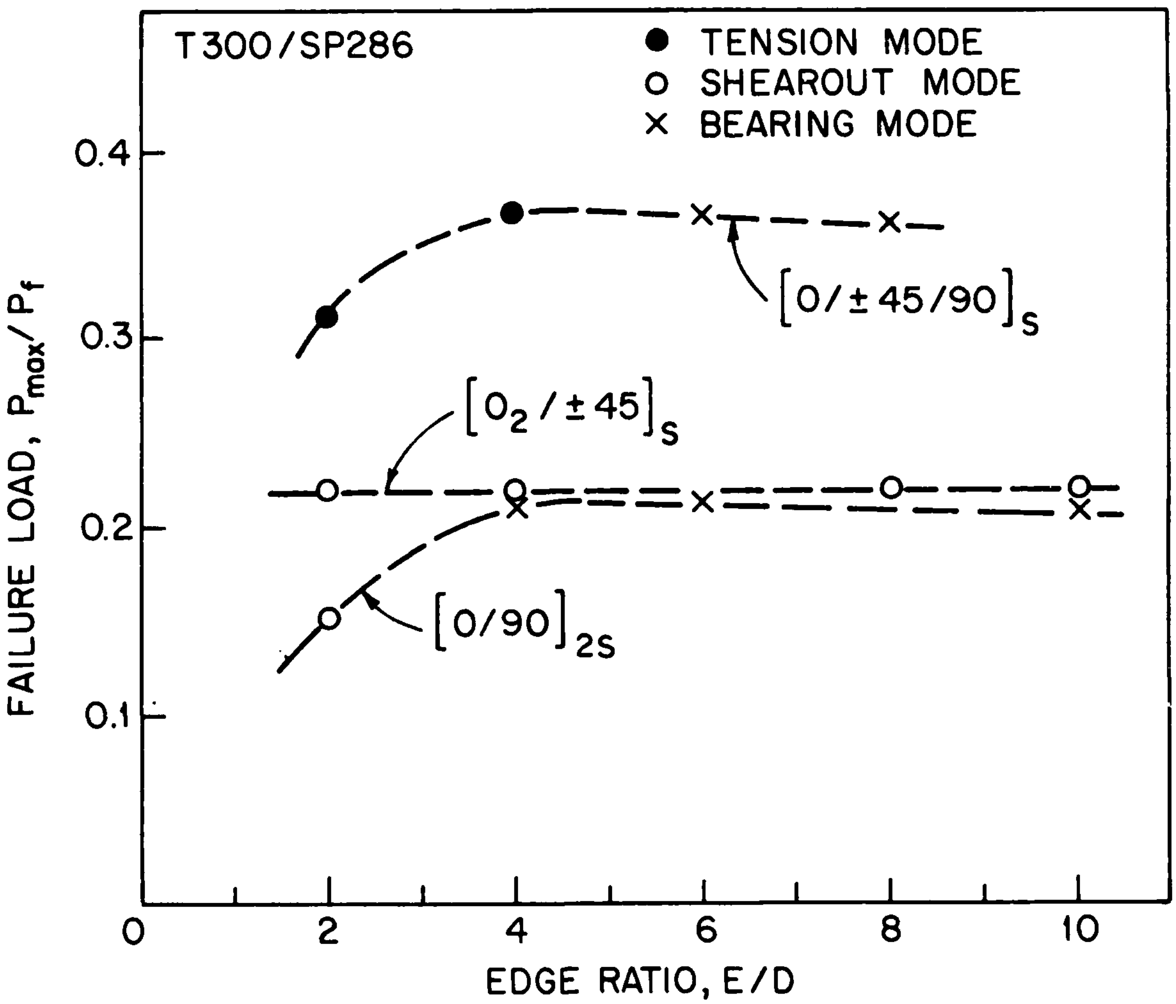

Figure 12. The effects of edge ratio on the failure load of laminates with different ply orientations. Parameters used in the numerical calculations: Material: Graphite/Epoxy T300/SP286, D=5.08 mm, W/D=5, L/D=14

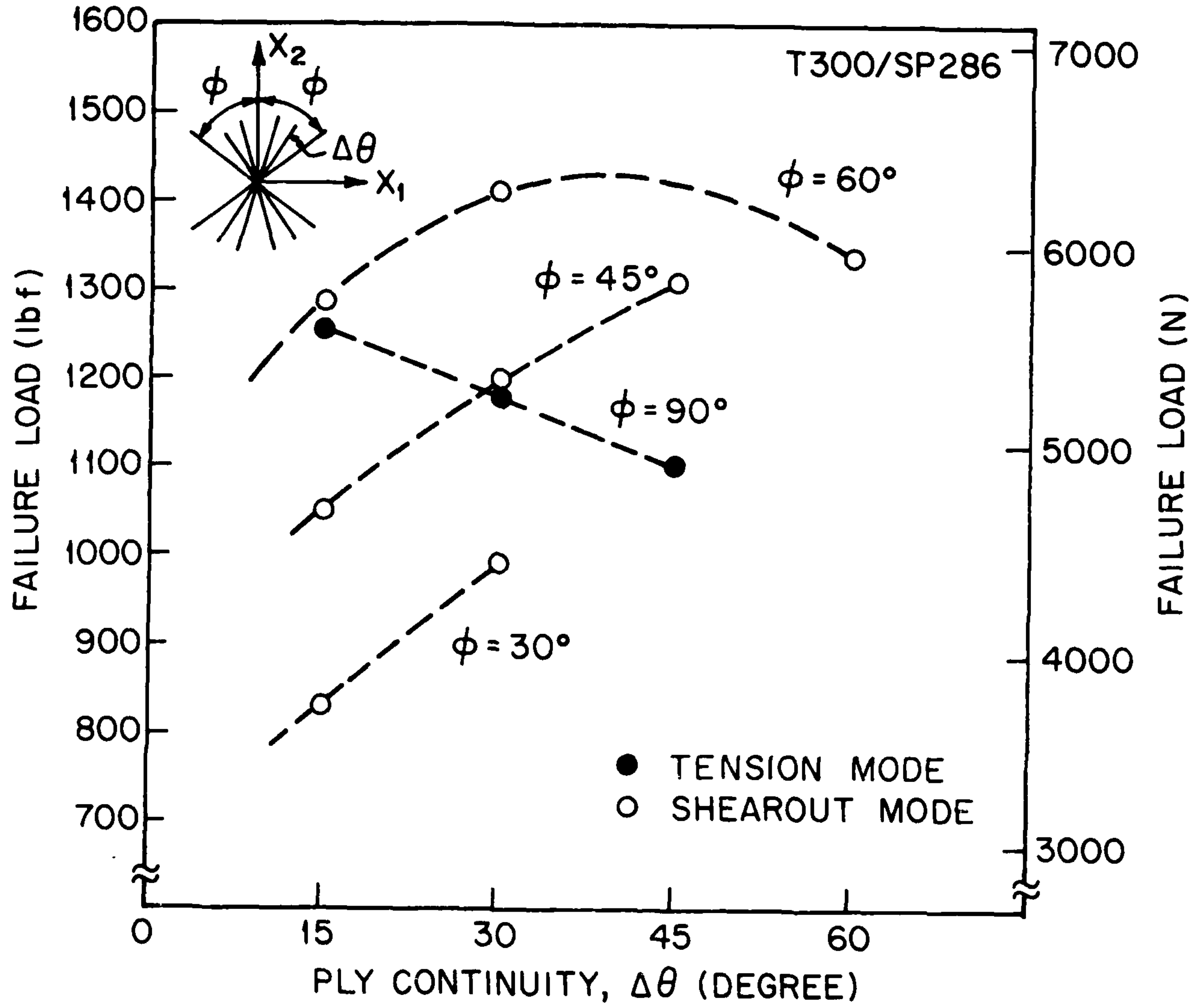

Figure 13. The effects of maximum ply angle ϕ and ply continuity $\Delta\theta$ on the failure load of mechanically fastened joints. Parameters used in the numerical calculations: Material: Graphite/Epoxy T300/SP286, D=4.76 mm, W/D=5.336, E/D=2.983, L/D=14.68, H=1.397 mm

orientation between two adjacent plies $\Delta\theta$. The latter parameter is referred to here as "ply continuity". The results in Figure 13 show that the failure load increases both with increasing ϕ and with increasing $\Delta\theta$, as long as failure is by shearout mode. On the other hand, the failure load decreases with increasing ϕ and with increasing $\Delta\theta$ when the failure is by tension mode. These results indicate that care must be exercised in designing bolted joints. If there are no other design constraints, the range of ply orientation ϕ and the ply continuity $\Delta\theta$ should be determined with the use of the computer code such that the joint can withstand the highest load.

SECTION VIII

CONCLUDING REMARKS

The model and computer code developed in this investigation can be used in the design of mechanically fastened joints involving fiber reinforced laminates. The computer code can be used to determine

a) the optimum geometry of a joint for a given load,

b) whether or not the joint will fail under a given load,

c) the failure load,

d) the mode of failure, and

e) the ply in which failure first occurs.

The good accuracy of the method suggests that it might be worth it to extend the method to joints consisting of two or more fasteners.

The results of parametric studies performed with the present computer code show that the material properties, joint geometry, and ply orientation, all effect significantly the strength of mechanically fastened joints.

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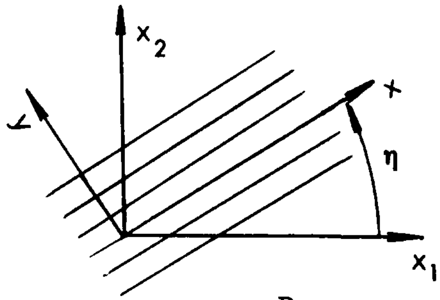
Appendix A - The Transformed Reduced Stiffness Matrix Q_{ij}



The components of the matrix $\bar{Q}_{ij}^P$ appearing in Eq. (16) are

$$\bar{Q}_{11}^P = Q_{11}^P\cos^4\eta+2(Q_{12}^P+2Q_{66}^P)\sin^2\eta\cos^2\eta+Q_{22}^P\sin^4\eta$$
$$\bar{Q}_{12}^P = (Q_{11}^P+Q_{22}^P-4Q_{66}^P)\sin^2\eta\cos^2\eta+Q_{12}^P(\sin^4\eta+\cos^4\eta)$$
$$\bar{Q}_{22}^P = Q_{11}^P\sin^4\eta+2(Q_{12}^P+2Q_{66}^P)\sin^2\eta\cos^2\eta+Q_{22}^P\cos^4\eta$$
$$\bar{Q}_{13}^P = (Q_{11}^P-Q_{12}^P-2Q_{33}^P)\sin\eta\cos^3\eta+(Q_{12}^P-Q_{22}^P+2Q_{33}^P)\sin^3\eta\cos\eta$$
$$\bar{Q}_{23}^P = (Q_{11}^P-Q_{12}^P-2Q_{33}^P)\sin^3\eta\cos\eta+(Q_{12}^P-Q_{22}^P+2Q_{33}^P)\sin\eta\cos^3\eta$$
$$\bar{Q}_{33}^P = (Q_{11}^P+Q_{22}^P-2Q_{12}^P-2Q_{33}^P)\sin^2\eta\cos^2\eta+Q_{33}^P(\sin^4\eta+\cos^4\eta)$$

in which

$$Q_{11}^P = \frac{E_1^P}{1-\nu_{12}^P\nu_{21}^P}$$

$$Q_{12}^P \quad \frac{\nu_{12}^P\,E_2^P}{1-\nu_{12}^P\nu_{21}^P} = \frac{\nu_{21}^P\,E_1^P}{1-\nu_{12}^P\nu_{21}^P}$$

$$Q_{22}^P = \frac{E_2^P}{1-\nu_{12}^P\nu_{21}^P}$$

$$Q_{33}^P = G_{12}^P$$

The superscript p denotes the material properties of the p-th ply and the angle η is measured from the x_1-axis to the x-axis. E_1^P, E_2^P and G_{12}^P are the longitudinal, transverse and shear moduli of the p-th ply, respectively. ν_{12}^P and ν_{21}^P are Poisson's rations of p-th ply and satisfy the relation

$$\frac{\nu_{12}^P}{E_1^P} = \frac{\nu_{21}^P}{E_2^P}$$

Appendix B - Shape Function Used in the Finite Element Code.

In the isoparametric element, the geometry and the displacement of the element are described in terms of the shape function N_α by a transformation from a Master Element in the r-s coordinate system to the element in the x_1-x_2 coordinate system (Figure 14)

$$X_i = N_\alpha(r,s)\ \bar{x}_{i\alpha} \qquad i = 1,2$$

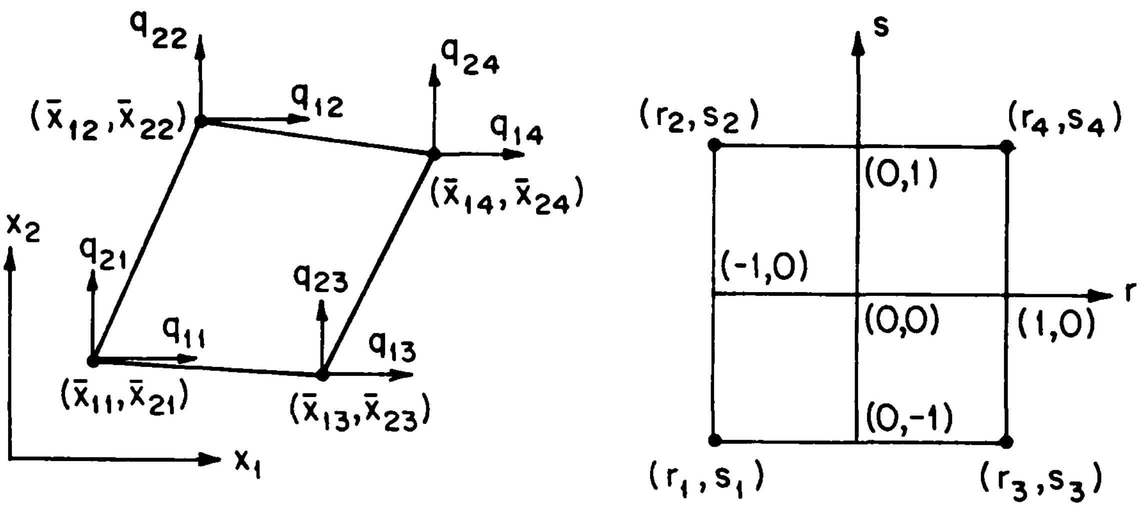
$$u_i = N_\alpha(r,s)\ q_{i\alpha} \qquad \alpha = 1,2,3 \text{ or } 4$$

$$N_\alpha(r,s) = 1/4(1+r\,r_\alpha)(1+ss_\alpha) \qquad -1 \leq r,s \leq 1$$

where $x_{i\alpha}$ is the coordinate of node α in the i-direction and $q_{i\alpha}$ is the displacement of node α in the i-direction, and r_α and s_α are the coordinates of node α referred to th Master Element

Note the property

$$N_\alpha(r_\beta,s_\beta) = \begin{cases} 1, & \text{if } \alpha=\beta \\ 0, & \text{if } \alpha\neq\beta \end{cases}$$



Element in $x_1 - x_2$ coordinates

Master element in local r-s coordinates

Figure 14. Geometry of an element used in the finite element calculations. Left: Element in the x_1-x_2 coordinate system. Right: Element (master element) in the local (r-s) coordinate system. $x_{i\alpha}$ is the coordinate of node α in the i direction, $q_{i\alpha}$ is the displacement of node α in the i direction and (r_α, s_α) are the coordinates of node α in the r-s coordinate system, i=1,2, α=1,2,3 or 4

Appendix C - Listing of the Computer Code "BOLT", and a Sample of Input and Output.

```
C
C ****************************************************************
C
C
C                          <  BOLT  >
C
C
C                THIS PROGRAN IS DEVELOPED FOR PREDICTING THE FAILURE
C         LOAD AND FAILURE MODE OF SINGLE FASTENER COMPOSITE JOINTS AS
C         WELL AS LAMINATES CONTAINING A OPEN HOLE (WITHOUT LOAD) .  A
C         DISPLACEMENT MODLE OF FINITE ELEMENT METHOD IS USED TO SOLVE
C         THE STRESSES  AROUND  THE BOUNDARY OF THE HOLE  AND THEN THE
C         FAILURE HYPOTHESIS-CHARACTERISTIC CURVE TOGETHER WITH YAMADA
C         FAILURE CRITERION  IS USED TO  PREDICT THE  FAILURE LOAD AND
C         FAILURE MODE.
C
C
C
C                        -------   FU-KUO CHANG    ----------
C                         APPLIED MECHANICS AT UNIVERSITY OF MICHIGAN
C                          ANN ARBOR, MICHIGAN, 48109
C
C
C
C
C ****************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
     1RAN
      COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
      COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
      COMMON / AFORE/ PF,DP,F(706)
      COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
      COMMON /ALEM/ NODXY,NELX,IJK(4,400)
      COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
     1             ,SI(4),RI(4),P(20),Q(20)
      COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
     1)
      COMMON /AMAX/ SK(706,90),R1(706)
C
C
      DIMENSION DIS(2,400)
      EQUIVALENCE (DIS(1),R1(1))
C
C
      CALL PGAUSS
      CALL INPUT
      CALL MATRL
      CALL FORMK
      CALL BOUND
C
   10 DO 5 I=1,NEQ
      F(I)=F(I)*PF
    5 CONTINUE
      CALL SOLVE
```

```
             C      DO 15 I=1,353                                                   52.000
             C      WRITE(6,400) I,(DIS(J,I),J=1,2)                                 53.000
             C 400  FORMAT(/,5X,'POINT=',I3,5X,'X-DIS',E15.8,5X,'Y-DIS',E15.8/)     54.000
             C  15  CONTINUE                                                        55.000
  0022              CALL STRESS(DIS)                                                56.000
  0023              CALL CFAIL(KT,MT,FAL)                                           57.000
  0024              IF(FAL .GT. 1.) GO TO 100                                       58.000
             C                                                                      59.000
  0025              TN=DSQRT(1.0/FAL)                                               60.000
  0026              PF=TN*PF*HI*4.0                                                 61.000
             C                                                                      62.000
  0027          50  CALL OUTPUT(KT,MT,FAL)                                          63.000
  0028              GO TO 11                                                        64.000
  0029         100  WRITE(6,111)                                                    65.000
  0030         111  FORMAT (/,5X,' ****** THE APPLIED LOAD IS TOO HIGH   ****',/)   66.000
  0031          11  STOP                                                            67.000
  0032              END                                                             68.000
   *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
   *OPTIONS IN EFFECT*  NAME = MAIN    , LINECNT =       57
   *STATISTICS*    SOURCE STATEMENTS =       32,PROGRAM SIZE =       798
   *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001             SUBROUTINE PGAUSS
0002             IMPLICIT REAL*8 (A-H,O-Z)
0003             COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
                1               ,SI(4),RI(4),P(20),Q(20)
0004             COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0005             LINT=2
0006             RG(1)=0.577350269187
0007             RG(2)=-RG(1)
0008             SG(1)=RG(1)
0009             SG(2)=RG(2)
           C
0010             WG(1)=1.
0011             WG(2)=1.
           C
0012             RI(1)=-1.0
0013             SI(1)=-1.0
0014             RI(2)=-1.0
0015             SI(2)= 1.0
0016             RI(3)= 1.0
0017             SI(3)=-1.0
0018             RI(4)=1.0
0019             SI(4)= 1.0
           C
0020             RETURN
0021             END
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = PGAUSS   , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       21,PROGRAM SIZE =       424
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001            SUBROUTINE INPUT
           C
           C
           C
           C ********************************************************************
           C
           C
           C       -----  INPUT INSTRUCTION
           C
           C       1) INDICATOR
           C
           C           1.1) TEST :  A INDICATOR TO SPECIFY THE PROBLEM
           C
           C                 TEST=0  :  OPEN HOLE (WITHOUT LOAD)
           C
           C                 TEST=1  :  LOADED HOLE (JOINT)
           C
           C           1.2) NRAN  :  A INDICATOR TO SPECIFY THE PLY ORIENTATION
           C
           C                 NRAN=0  :  MAXIMUM   PLY ANGLE (ANG) AND  PLY
           C                            CONTINUITY (DANG) AS INPUT PARAMETERS
           C
           C                 NRAN=1  :  INDIVIDAL PLY ANGLE (ANT(I)) AS INPUT
           C                            PARAMETERS
           C
           C       2)  MATERIAL PROPERTIES
           C
           C             2.1) E1   :  PLY LONGITUDINAL TENSIL MODULOUS
           C
           C             2.2) V12  :  POISSON'S RATIO FOR TRANSVERSE STRAIN IN
           C                          THE Y-DIRECTION WHEN STRESSED IN THE
           C                          X-DIRECTION
           C
           C             2.3) G12  :  PLY SHEAR MODULUS
           C
           C             2.4) XX   :  PLY LONGITUDINAL TENSILE STRENGTH
           C
           C             2.5) SS   :  LAMINATE SHEAR STRENGTH OF CROSS-PLY
           C
           C             2.6) RT   :  CHARACTERISTIC LENGTH FOR TENSION
           C
           C             2.7) RC   :  CHARACTERISTIC LENGTH FOR COMPRESSION
           C
           C       3)  PLY ORIENTATION
           C
           C            3.1) ANG   :  MAXIMUM PLY ANGLE  (NRAN=0)
           C
           C            3.2) DANG  :  PLY CONTINUITY  (NRAN=0)
           C
           C            3.3) ANT(I)  :  INDIVIDUAL PLY ANGLE  (NRAN=1)
           C
           C       4)  GEOMETRY
           C
           C            4.1) D  :  DIAMETER
           C
```

```
C             4.2) WD  : WIDTH RATIO W/D
C
C             4.3) ED  :  EDGE RATIO E/D
C
C             4.4) LD  :  LENGTH RATIO L/D
C
C             4.5) NLY  :  A HALF NUMBER OF TOTAL PLIES IN THE
C                          SYMMETRIC LAMINATE
C
C             4.6) HI  :  ONE HALF OF LAMINATE THICKNESS
C
C             4.7) THICK(I)  :  THICKNESS OF I-TH PLY  (NRAN=1)
C
C
C
C       * NOTE  :
C                 IF  NRAN=0. NLY. THICK(I)'AND ANT(I) DO NOT
C                 HAVE TO BE INPUT.
C
C                 IF NRAN=1. INPUT ANG=0.0 AND DANG=1.0.
C
C
C
C
C ***********************************************************************
C *********************
C
C
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
     1RAN
      COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
      COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
      COMMON / AFORE/ PF,DP,F(706)
      COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
      COMMON /ALEM/ NODXY,NELX,IJK(4,400)
      COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
     1              ,SI(4),RI(4),P(20),Q(20)
      READ (5,1) ITEST,NRAN
    1 FORMAT (I5)
      READ (5,10) ANG,DANG
      READ (5,10) E1,V12,G12
      READ (5,10) D,WD,ED,ALD
      READ (5,10) XX,SS,RT,RC
      READ (5,10) HI
C
C
      IF(NRAN .EQ. 0) GO TO 6
      READ (5,1) NLY
      READ (5,10) (ANT(I),THICK(I),I=1,NLY)
   10 FORMAT (4F15.5)
C
    6 CONTINUE
      NP=4
```

```
0023          NX=353
0024          NELX=306
0025          NEQ=706
0026          NBAND=82
0027          E2=100
0028          PF=500
        C
        C
        C
0029          W=WD*D
0030          E=ED*D
0031          AL=ALD*D
0032          TP=4.0*PF*HI
        C
0033          CALL AMESH
        C
        C  ****  INPUT FIXED BOUNDARY POINTS  *****
        C
0034          NBC(1)=353
0035          NBC(2)=352
0036          NBC(3)=351
0037          NBC(4)=350
0038          NBC(5)=349
0039          NBC(6)=348
0040          NBC(7)=347
0041          NBC(8)=346
0042          NBC(9)=345
0043          NBC(10)=344
0044          NBC(11)=343
0045          NBC(12)=342
0046          NBC(13)=341
0047          NBC(14)=340
0048          NBC(15)=339
0049          NBC(16)=1
0050          NBC(17)=2
0051          NBC(18)=3
0052          NBC(19)=4
0053          NBC(20)=5
0054          NBC(21)=6
0055          NBC(22)=7
0056          NBC(23)=8
0057          NBC(24)=9
0058          NBC(25)=10
0059          NBC(26)=11
0060          IF(ITEST .EQ. 0) GO TO 40
0061          NBC(27)=338
0062          NBC(28)=323
0063          NBC(29)=308
0064          NBC(30)=290
0065          NBC(31)=291
0066          NBC(32)=292
0067          NBC(33)=293
        C
0068          IF(ITEST .EQ. 1) GO TO 50
        C
```

```
0069          40 NBC(27)=22                         255.000
0070             NBC(28)=33                         256.000
0071             NBC(29)=44                         257.000
0072             NBC(30)=55                         258.000
0073             NBC(31)=66                         259.000
0074             NBC(32)=77                         260.000
0075             NBC(33)=88                         261.000
0076             NBC(34)=99                         262.000
0077             NBC(35)=103                        263.000
0078             NBC(36)=107                        264.000
0079             NBC(37)=111                        265.000
            C                                       266.000
            C                                       267.000
0080          50 NFIX(1)=11                         268.000
0081             IF(ITEST .EQ. 0) NFIX(1)=10        269.000
0082             NFIX(2)=10                         270.000
0083             NFIX(3)=10                         271.000
0084             NFIX(4)=10                         272.000
0085             NFIX(5)=10                         273.000
0086             NFIX(6)=10                         274.000
0087             NFIX(7)=10                         275.000
0088             NFIX(8)=10                         276.000
0089             NFIX(9)=10                         277.000
0090             NFIX(10)=10                        278.000
0091             NFIX(11)=10                        279.000
0092             NFIX(12)=10                        280.000
0093             NFIX(13)=10                        281.000
0094             NFIX(14)=10                        282.000
0095             NFIX(15)=10                        283.000
0096             NFIX(16)=10                        284.000
            C                                       285.000
            C    IF(ITEST .EQ. 1) NFIX(16)=11       286.000
            C                                       287.000
0097             NFIX(17)=10                        288.000
0098             NFIX(18)=10                        289.000
0099             NFIX(19)=10                        290.000
0100             NFIX(20)=10                        291.000
0101             NFIX(21)=10                        292.000
0102             NFIX(22)=10                        293.000
0103             NFIX(23)=10                        294.000
0104             NFIX(24)=10                        295.000
0105             NFIX(25)=10                        296.000
0106             NFIX(26)=11                        297.000
            C                                       298.000
0107             IF(ITEST .EQ. 1) NFIX(26)=10       299.000
            C                                       300.000
            C                                       301.000
0108             NFIX(27)=1                         302.000
0109             NFIX(28)=1                         303.000
0110             NFIX(29)=1                         304.000
0111             NFIX(30)=1                         305.000
0112             NFIX(31)=1                         306.000
0113             NFIX(32)=1                         307.000
0114             NFIX(33)=1                         308.000
0115             IF(ITEST .EQ. 1) GO TO 55          309.000
```

```
0116          NFIX(34)=1
0117          NFIX(35)=1
0118          NFIX(36)=1
0119          NFIX(37)=1
        C
        C   *****  INPUT  NODAL UNIT FORCES  ************
        C
0120       55 FT=X(262)/(W/2.0)
0121          FT3=X(316)/(W/2.0)
0122          FT2=FT-FT3
0123          FT1=1-FT
        C
0124          DO 100 I=1,706
0125          F(I)=0.0
0126      100 CONTINUE
0127          IF(ITEST .EQ. 1) GO TO 155
        C
0128          F(586)=-FT1/6.0
0129          F(584)=-FT1/3.0
0130          F(582)=-FT1/3.0
0131          F(580)=-(FT1/6.0)-(FT2/4.0)
0132          F(616)=-FT2/2.0
0133          F(646)=-(FT2/4.0)-(FT3/4.0)
0134          F(676)=-FT3/2.
0135          F(706)=-FT3/4.0
0136          IF(ITEST .EQ. 0) GO TO 150
        C
0137      155 PI=3.141596535
0138          F(1)=2.0*(4.0/(D*PI))*0.0
0139          F(2)=0.062499988
0140          F(23)=0.012193140
0141          F(24)=0.12379906
0142          F(45)=0.023917705
0143          F(46)=0.12024245
0144          F(67)=0.034723127
0145          F(68)=0.11446683
0146          F(89)=0.044194158
0147          F(90)=0.10669416
0148          F(111)=0.051966834
0149          F(112)=0.097223130
0150          F(133)=0.057742454
0151          F(134)=0.086417711
0152          F(155)=0.061299064
0153          F(156)=0.074693147
0154          F(177)=0.062499988
0155          F(178)=0.062500007
0156          F(223)=0.061299072
0157          F(224)=0.050306867
0158          F(245)=0.057742469
0159          F(246)=0.038582300
0160          F(267)=0.051966855
0161          F(268)=0.027776877
0162          F(289)=0.044194186
0163          F(290)=0.018305843
0164          F(311)=0.034723159
```

```
 0165              F(312)=0.010533165                                                      365.000
 0166              F(333)=0.023917741                                                      366.000
 0167              F(334)=0.0047575414                                                     367.000
 0168              F(355)=0.012193179                                                      368.000
 0169              F(356)=0.0012009269                                                     369.000
 0170              F(377)=0.0020453255                                                     370.000
 0171              F(378)=2.0*0.00                                                         371.000
            C                                                                              372.000
            C                                                                              373.000
 0172         150 CONTINUE                                                                 374.000
            C                                                                              375.000
 0173              RETURN                                                                  376.000
 0174              END                                                                     377.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = INPUT   , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =      174,PROGRAM SIZE =     2894
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
      SUBROUTINE AMESH
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
     1RAN
      COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
      COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
      COMMON /ALEM/ NODXY,NELX,IJK(4,400)
      COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
     1                  ,SI(4),RI(4),P(20),Q(20)
      DIMENSION BX(33,10),BY(33,10)
      DIMENSION NES(30),NS(30),K12(30),K43(30),K4(30),MX(30),MY(30),KY(3
     10),KSC(30)
      DIMENSION SH(8)
C
C       ******  READ INPUT DATA FOR THE GLOBAL MODEL*****
C
C
C
      TH=0.09817477
      Z=W/3.6
      ZN=Z-(D/2.0)
C
C
      BX(1,1)=0.0
      BY(1,1)=D/2.
      TH2=TH*2
      TH4=TH*4
      TH6=TH*6
      TH8=TH*8
      TH10=TH*10
      TH12=TH*12
      TH14=TH*14
      BX(1,5)=D/2.*DSIN(TH2)
      BY(1,5)=D/2.*DCOS(TH2)
      BX(1,2)=D/2.*DSIN(TH4)
      BY(1,2)=D/2.*DCOS(TH4)
      BX(1,6)=FTH(TH4)*DSIN(TH4)
      BY(1,6)=FTH(TH4)*DCOS(TH4)
      BX(1,3)=(D/2.+ZN)/DCOS(TH4)*DSIN(TH4)
      BY(1,3)=(D/2.+ZN)
      BX(1,7)=(D/2.+ZN)/DCOS(TH2)*DSIN(TH2)
      BY(1,7)=(D/2.+ZN)
      BX(1,4)=0.0
      BY(1,4)=(D/2.+ZN)
      BX(1,8)=0.0
      BY(1,8)=D/2.+ZN/2.0
C
C
      BX(2,1)=0.0
      BY(2,1)=(D/2.)+ZN
      BX(2,2)=BX(1,3)
      BY(2,2)=BY(1,3)
      BX(2,3)=BX(2,2)
      BY(2,3)=E
      BX(2,4)=0.0
```

```
0044          BY(2,4)=E
           C
           C
0045          BX(3,1)=BX(1,2)
0046          BY(3,1)=BY(1,2)
0047          BX(3,5)=D/2.*DSIN(TH6)
0048          BY(3,5)=D/2.*DCOS(TH6)
0049          BX(3,2)=D/2.*DSIN(TH8)
0050          BY(3,2)=D/2.*DCOS(TH8)
0051          BX(3,6)=FTH(TH8)*DSIN(TH8)
0052          BY(3,6)=FTH(TH8)*DCOS(TH8)
0053          BX(3,3)=((D/2.+ZN)/DCOS(TH8))*DSIN(TH8)
0054          BY(3,3)=(D/2.)+ZN
0055          BX(3,7)=(D/2.+ZN)/DCOS(TH6)*DSIN(TH6)
0056          BY(3,7)=(D/2.)+ZN
0057          BX(3,4)=BX(1,3)
0058          BY(3,4)=BY(1,3)
0059          BX(3,8)=BX(1,6)
0060          BY(3,8)=BY(1,6)
           C
           C
0061          BX(4,1)=BX(1,3)
0062          BY(4,1)=BY(1,3)
0063          BX(4,2)=BX(3,3)
0064          BY(4,2)=BY(3,3)
0065          BX(4,3)=BX(4,2)
0066          BY(4,3)=E
0067          BX(4,4)=BX(2,3)
0068          BY(4,4)=BY(2,3)
           C
           C
0069          BX(5,1)=BX(3,3)
0070          BY(5,1)=BY(3,3)
0071          BX(5,2)=W/2.0
0072          BY(5,2)=BY(3,3)
0073          BX(5,3)=W/2.0
0074          BY(5,3)=BY(2,4)
0075          BX(5,4)=BX(4,3)
0076          BY(5,4)=BY(4,3)
           C
           C
0077          BX(6,1)=BX(3,2)
0078          BY(6,1)=BY(3,2)
0079          BX(6,5)=D/2.*DSIN(TH10)
0080          BY(6,5)=D/2.*DCOS(TH10)
0081          BX(6,2)=D/2.*DSIN(TH12)
0082          BY(6,2)=D/2.*DCOS(TH12)
0083          BX(6,6)=FTH(TH4)*DSIN(TH12)
0084          BY(6,6)=FTH(TH4)*DCOS(TH12)
0085          BX(6,3)=(D/2.)+ZN
0086          BY(6,3)=BX(6,3)/DCOS(TH4)*DSIN(TH4)
0087          BX(6,7)=(D/2.)+ZN
0088          BY(6,7)=BX(6,7)/DCOS(TH6)*DSIN(TH6)
0089          BX(6,4)=BX(3,3)
0090          BY(6,4)=BY(3,3)
```

```
0091           BX(6,8)=BX(3,6)
0092           BY(6,8)=BY(3,6)
          C
          C
0093           BX(7,1)=D/2.+ZN
0094           BY(7,1)=((BY(6,4)-BY(6,3))/4.)*3.+BY(6,3)
0095           BX(7,2)=BX(6,3)
0096           BY(7,2)=BY(6,3)
0097           BX(7,3)=W/2.0
0098           BY(7,3)=BY(6,3)
0099           BX(7,4)=W/2.0
0100           BY(7,4)=BY(7,1)
          C
          C
0101           BX(8,1)=BX(6,2)
0102           BY(8,1)=BY(6,2)
0103           BX(8,5)=D/2.*DSIN(TH14)
0104           BY(8,5)=D/2.*DCOS(TH14)
0105           BX(8,2)=D/2.
0106            BY(8,2)=0.0
0107           BX(8,6)=D/2.+ZN/2.0
0108           BY(8,6)=0.0
0109           BX(8,3)=(D/2.)+ZN
0110           BY(8,3)=0.0
0111           BX(8,7)=(D/2.)+ZN
0112           BY(8,7)=BX(8,7)/DCOS(TH2)*DSIN(TH2)
0113           BX(8,4)=BX(6,3)
0114           BY(8,4)=BY(6,3)
0115           BX(8,8)=BX(6,6)
0116           BY(8,8)=BY(6,6)
          C
          C
0117           BX(9,1)=BX(8,4)
0118           BY(9,1)=BY(8,4)
0119           BX(9,2)=BX(8,3)
0120           BY(9,2)=BY(8,3)
0121           BX(9,3)=W/2.0
0122           BY(9,3)=0.0
0123           BX(9,4)=W/2.0
0124           BY(9,4)=BY(8,4)
          C
          C
0125           BX(10,1)=D/2.
0126           BY(10,1)=0.0
0127           BX(10,5)=BX(8,5)
0128           BY(10,5)=-1*BY(8,5)
0129           BX(10,2)=BX(8,1)
0130           BY(10,2)=-BY(8,1)
0131           BX(10,6)=BX(8,8)
0132           BY(10,6)=-BY(8,8)
0133           BX(10,3)=BX(8,4)
0134           BY(10,3)=-BY(8,4)
0135           BX(10,7)=BX(8,7)
0136           BY(10,7)=-BY(8,7)
0137           BX(10,4)=BX(8,3)
```

```
0138          BY(10,4)=-BY(8,3)          541.000
0139          BX(10,8)=BX(8,6)           542.000
0140          BY(10,8)=-BY(8,6)          543.000
        C                                544.000
        C                                545.000
0141          BX(11,1)=BX(9,2)           546.000
0142          BY(11,1)=-BY(9,2)          547.000
0143          BX(11,2)=BX(9,1)           548.000
0144          BY(11,2)=-BY(9,1)          549.000
0145          BX(11,3)=BX(9,4)           550.000
0146          BY(11,3)=-BY(9,4)          551.000
0147          BX(11,4)=BX(9,3)           552.000
0148          BY(11,4)=-BY(9,3)          553.000
        C                                554.000
        C                                555.000
0149          BX(12,1)=BX(6,2)           556.000
0150          BY(12,1)=-BY(6,2)          557.000
0151          BX(12,5)=BX(6,5)           558.000
0152          BY(12,5)=-BY(6,5)          559.000
0153          BX(12,2)=BX(6,1)           560.000
0154          BY(12,2)=-BY(6,1)          561.000
0155          BX(12,6)=BX(6,8)           562.000
0156          BY(12,6)=-BY(6,8)          563.000
0157          BX(12,3)=BX(6,4)           564.000
0158          BY(12,3)=-BY(6,4)          565.000
0159          BX(12,7)=BX(6,7)           566.000
0160          BY(12,7)=-BY(6,7)          567.000
0161          BX(12,4)=BX(6,3)           568.000
0162          BY(12,4)=-BY(6,3)          569.000
0163          BX(12,8)=BX(6,6)           570.000
0164          BY(12,8)=-BY(6,6)          571.000
        C                                572.000
        C                                573.000
0165          BX(13,1)=BX(7,2)           574.000
0166          BY(13,1)=-BY(7,2)          575.000
0167          BX(13,2)=BX(6,4)           576.000
0168          BY(13,2)=-BY(6,4)          577.000
0169          BX(13,3)=W/2.0             578.000
0170          BY(13,3)=-(D/2.+ZN)        579.000
0171          BX(13,4)=BX(7,3)           580.000
0172          BY(13,4)=-BY(7,3)          581.000
        C                                582.000
        C                                583.000
0173          BX(14,1)=BX(13,2)          584.000
0174          BY(14,1)=BY(13,2)          585.000
0175          BX(14,2)=BX(13,2)          586.000
0176          BY(14,2)=-(AL-E)           587.000
0177          BX(14,3)=W/2.0             588.000
0178          BY(14,3)=BY(14,2)          589.000
0179          BX(14,4)=W/2.0             590.000
0180          BY(14,4)=BY(13,3)          591.000
        C                                592.000
        C                                593.000
0181          BX(15,1)=BX(3,2)           594.000
0182          BY(15,1)=-BY(3,2)          595.000
```

```
0183          BX(15,5)=BX(3,5)                              596.000
0184          BY(15,5)=-BY(3,5)                             597.000
0185          BX(15,2)=BX(3,1)                              598.000
0186          BY(15,2)=-BY(3,1)                             599.000
0187          BX(15,6)=BX(3,8)                              600.000
0188          BY(15,6)=-BY(3,8)                             601.000
0189          BX(15,3)=BX(3,4)                              602.000
0190          BY(15,3)=-BY(3,4)                             603.000
0191          BX(15,7)=BX(3,7)                              604.000
0192          BY(15,7)=-BY(3,7)                             605.000
0193          BX(15,4)=BX(3,3)                              606.000
0194          BY(15,4)=-BY(3,3)                             607.000
0195          BX(15,8)=BX(3,6)                              608.000
0196          BY(15,8)=-BY(3,6)                             609.000
         C                                                  610.000
         C                                                  611.000
0197          BX(16,1)=(BX(15,4)-BX(15,3))/2.+BX(15,3)      612.000
0198          BY(16,1)=-(D/2.+ZN)                           613.000
0199          BX(16,2)=BX(15,3)                             614.000
0200          BY(16,2)=-(D/2.+ZN)                           615.000
0201          BX(16,3)=BX(15,3)                             616.000
0202          BY(16,3)=-(AL-E)                              617.000
0203          BX(16,4)=BX(16,1)                             618.000
0204          BY(16,4)=-(AL-E)                              619.000
         C                                                  620.000
         C                                                  621.000
0205          BX(17,1)=BX(1,2)                              622.000
0206          BY(17,1)=-BY(1,2)                             623.000
0207          BX(17,5)=BX(1,5)                              624.000
0208          BY(17,5)=-BY(1,5)                             625.000
0209          BX(17,2)=BX(1,1)                              626.000
0210          BY(17,2)=-BY(1,1)                             627.000
0211          BX(17,6)=BX(1,8)                              628.000
0212          BY(17,6)=-BY(1,8)                             629.000
0213          BX(17,3)=BX(1,4)                              630.000
0214          BY(17,3)=-BY(1,4)                             631.000
0215          BX(17,7)=BX(1,7)                              632.000
0216          BY(17,7)=-BY(1,7)                             633.000
0217          BX(17,4)=BX(1,3)                              634.000
0218          BY(17,4)=-BY(1,3)                             635.000
0219          BX(17,8)=BX(1,6)                              636.000
0220          BY(17,8)=-BY(1,6)                             637.000
         C                                                  638.000
         C                                                  639.000
0221          BX(18,1)=BX(17,4)                             640.000
0222          BY(18,1)=BY(17,4)                             641.000
0223          BX(18,2)=0.0                                  642.000
0224          BY(18,2)=BY(15,4)                             643.000
0225          BX(18,3)=0.0                                  644.000
0226          BY(18,3)=-(AL-E)                              645.000
0227          BX(18,4)=BX(17,4)                             646.000
0228          BY(18,4)=-(AL-E)                              647.000
         C                                                  648.000
0229          NBLOCK=18                                     649.000
0230          NES(1)=0                                      650.000
```

```
0231          NES(2)=28
0232          NES(3)=40
0233          NES(4)=68
0234          NES(5)=80
0235          NES(6)=89
0236          NES(7)=117
0237          NES(8)=126
0238          NES(9)=154
0239          NES(10)=166
0240          NES(11)=194
0241          NES(12)=206
0242          NES(13)=220
0243          NES(14)=226
0244          NES(15)=247
0245          NES(16)=261
0246          NES(17)=268
0247          NES(18)=282
        C
        C
0248          NS(1)=0
0249          NS(2)=7
0250          NS(3)=44
0251          NS(4)=51
0252          NS(5)=95
0253          NS(6)=88
0254          NS(7)=118
0255          NS(8)=144
0256          NS(9)=151
0257          NS(10)=188
0258          NS(11)=195
0259          NS(12)=232
0260          NS(13)=239
0261          NS(14)=261
0262          NS(15)=254
0263          NS(16)=300
0264          NS(17)=308
0265          NS(18)=315
        C
        C
0266          DO 11 I=1,18
0267       11 K12(I)=0
        C
0268          K12(2)=7
0269          K12(4)=7
0270          K12(7)=7
0271          K12(9)=7
0272          K12(11)=7
0273          K12(13)=7
0274          K12(16)=7
0275          K12(18)=7
        C
        C
0276          DO 12 I=1,18
0277       12 K43(I)=0
        C
```

```
0278              K43(1)=3
0279              K43(3)=3
0280              K43(6)=3
0281              K43(8)=3
0282              K43(10)=3
0283              K43(12)=3
0284              K43(15)=7
0285              K43(17)=7
           C
           C
0286              K2=0
           C
           C
0287              DO 13 I=1,18
0288           13 K4(I)=0
           C
0289              K4(6)=12
0290              K4(15)=24
           C
           C
0291              DO 14 I=1,18
0292           14 MX(I)=4
           C
0293              MX(5)=3
0294              MX(7)=3
0295              MX(12)=2
0296              MX(13)=2
0297              MX(14)=7
0298              MX(15)=2
0299              MX(16)=1
0300              MX(17)=2
0301              MX(18)=2
           C
           C
0302              DO 15 I=1,18
0303           15 MY(I)=7
           C
           C
0304              MY(2)=3
0305              MY(4)=3
0306              MY(5)=3
0307              MY(7)=3
0308              MY(9)=3
0309              MY(11)=3
0310              MY(13)=3
0311              MY(14)=3
           C
           C
0312              KX=1
           C
           C
0313              DO 16 I=1,18
0314           16 KY(I)=2
0315              KY(2)=1
0316              KY(4)=1
```

```
0317          KY(5)=1
0318          KY(7)=1
0319          KY(9)=1
0320          KY(11)=1
0321          KY(13)=1
0322          KY(14)=1
0323          KY(16)=1
0324          KY(18)=1
        C
        C
0325          DO 17 I=1,18
0326       17 KSC(I)=4
        C
0327          KSC(1)=8
0328          KSC(3)=8
0329          KSC(6)=8
0330          KSC(8)=8
0331          KSC(10)=8
0332          KSC(12)=8
0333          KSC(15)=8
0334          KSC(17)=8
        C
        C
        C
0335          DO 300 I=1,NBLOCK
        C
        C
        C
        C    *********COORDINATE  ***********
        C
0336          MXX=MX(I)
0337          MYY=MY(I)
0338          MXX1=MXX+1
0339          MYY1=MYY+1
0340          DO 100 IX=1,MXX1
0341          IF(IX.EQ.1) NSI=NS(I)
0342          IF(IX.GT.1.AND.IX.LT.MXX1) NSI=NS(I)+K4(I)+(IX-1)*(MYY1+K12(I)+K43
             1(I))
0343          IF(IX.EQ.MXX1) NSI=NS(I)+K4(I)+K2+(IX-1)*(MYY1+K12(I)+K43(I))
0344          XLI=FUN1(IX,MXX,KX)
0345          DO 100 IY=1,MYY1
0346          KY1=KY(I)
0347          YLI=FUN1(IY,MYY,KY1)
0348          CONTINUE
0349          IF(KSC(I).EQ.4) CALL SEREND(SH,XLI,YLI,4)
0350          IF(KSC(I).EQ.8) CALL SEREND(SH,XLI,YLI,8)
0351          XI=0.
0352          YI=0.
0353          KSC1=KSC(I)
0354          DO 102 K=1,KSC1
0355          SHK=SH(K)
0356          XI=XI+BX(I,K)*SHK
0357      102 YI=YI+BY(I,K)*SHK
0358          NIXY=NSI+IY
0359          X(NIXY)=XI
```

```
 0360              Y(NIXY)=YI                                                 815.000
 0361          100 CONTINUE                                                   816.000
             C                                                                817.000
             C                                                                818.000
             C     ******   ELEMENT  ********                                 819.000
             C                                                                820.000
             C       ***** 4-NODE ELEMENT  *******                            821.000
 0362              DO 202 IX=1,MXX                                            822.000
 0363              NSI=NS(I)+(IX-1)*(MYY1+K12(I)+K43(I))                      823.000
 0364              DO 202 IY=1,MYY                                            824.000
 0365              NN=NSI+IY                                                  825.000
 0366              NEL=NES(I)+(IX-1)*MY(I)+IY                                 826.000
 0367              IJK(1,NEL)=NN+K4(I)                                        827.000
 0368              IF(IX.EQ.1) IJK(1,NEL)=NN                                  828.000
 0369              IJK(3,NEL)=NN+K4(I)+(MYY1+K12(I)+K43(I))                   829.000
 0370              IJK(4,NEL)=IJK(3,NEL)+1                                    830.000
 0371              IJK(2,NEL)=IJK(1,NEL)+1                                    831.000
 0372              IF(IX.NE.MXX) GO TO 202                                    832.000
 0373              IJK(2,NEL)=IJK(2,NEL)+K2                                   833.000
 0374              IJK(3,NEL)=IJK(3,NEL)+K2                                   834.000
 0375          202 CONTINUE                                                   835.000
             C                                                                836.000
 0376          206 CONTINUE                                                   837.000
             C                                                                838.000
 0377          300 CONTINUE                                                   839.000
 0378              IJK(1,297)=96                                              840.000
 0379              IJK(2,297)=100                                             841.000
 0380              IJK(3,297)=119                                             842.000
 0381              IJK(4,297)=120                                             843.000
 0382              IJK(1,298)=100                                             844.000
 0383              IJK(2,298)=104                                             845.000
 0384              IJK(3,298)=120                                             846.000
 0385              IJK(4,298)=121                                             847.000
 0386              IJK(1,299)=104                                             848.000
 0387              IJK(2,299)=108                                             849.000
 0388              IJK(3,299)=121                                             850.000
 0389              IJK(4,299)=122                                             851.000
 0390              IJK(1,300)=262                                             852.000
 0391              IJK(2,300)=266                                             853.000
 0392              IJK(3,300)=301                                             854.000
 0393              IJK(4,300)=302                                             855.000
 0394              IJK(1,301)=266                                             856.000
 0395              IJK(2,301)=270                                             857.000
 0396              IJK(3,301)=302                                             858.000
 0397              IJK(4,301)=303                                             859.000
 0398              IJK(1,302)=270                                             860.000
 0399              IJK(2,302)=274                                             861.000
 0400              IJK(3,302)=303                                             862.000
 0401              IJK(4,302)=304                                             863.000
 0402              IJK(1,303)=274                                             864.000
 0403              IJK(2,303)=278                                             865.000
 0404              IJK(3,303)=304                                             866.000
 0405              IJK(4,303)=305                                             867.000
 0406              IJK(1,304)=278                                             868.000
 0407              IJK(2,304)=282                                             869.000
```

```
 0408             IJK(3,304)=305                                          870.000
 0409             IJK(4,304)=306                                          871.000
 0410             IJK(1,305)=282                                          872.000
 0411             IJK(2,305)=286                                          873.000
 0412             IJK(3,305)=306                                          874.000
 0413             IJK(4,305)=307                                          875.000
 0414             IJK(1,306)=286                                          876.000
 0415             IJK(2,306)=290                                          877.000
 0416             IJK(3,306)=307                                          878.000
 0417             IJK(4,306)=308                                          879.000
            C     WRITE (6,900)                                           880.000
            C 900 FORMAT(//5X,'<COORDINATE>',/)                           881.000
            C     WRITE(6,902) (I,X(I),Y(I),I=1,NX)                       882.000
            C 902 FORMAT(10X,I5,2X,2F10.3,5X,I5,2X,2F10.3)                883.000
            C     WRITE(6,904)                                            884.000
            C 904 FORMAT(//5X,'<ELEMENT>',/)                              885.000
            C     DO 906 NEL=1,NELX                                       886.000
            C 906 WRITE(6,908) NEL,(IJK(I,NEL),I=1,NP)                    887.000
            C 908 FORMAT(10X,I5,5X,9I5)                                   888.000
            C     WRITE(6,909)(P(I),Q(I),LEB(I),I=1,18)                   889.000
            C 909 FORMAT(//,5X,'P=',E15.6,5X,'Q=',E15.6,5X,'LEB=',I3/)    890.000
 0418             RETURN                                                  891.000
 0419             END                                                     892.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = AMESH    , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       419,PROGRAM SIZE =     13950
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001             SUBROUTINE SEREND(SH,XL,YL,KKK)
           C
           C
0002             IMPLICIT REAL*8(A-H,O-Z)
0003             DIMENSION SH(8)
           C
0004             K4=KKK/4
0005             GO TO (100,200),K4
           C      ***** 4-NODE ELEMENT  ******
0006         100 CONTINUE
0007             SH(1)=0.25*(1-XL)*(1.-YL)
0008             SH(2)=0.25*(1.+XL)*(1.-YL)
0009             SH(3)=0.25*(1.+XL)*(1.+YL)
0010             SH(4)=0.25*(1.-XL)*(1.+YL)
0011             RETURN
           C
           C    ***8-NODE ELEMENT ****
0012         200 CONTINUE
0013             SH(1)=-0.25*(XL+YL+1.)*(XL-1.)*(YL-1.)
0014             SH(2)=-0.25*(XL-YL-1.)*(XL+1.)*(YL-1.)
0015             SH(3)= 0.25*(XL+YL-1.)*(XL+1.)*(YL+1.)
0016             SH(4)= 0.25*(XL-YL+1.)*(XL-1.)*(YL+1.)
0017             SH(5)= 0.5*(XL*XL-1.)*(YL-1.)
0018             SH(6)=-0.5*(XL+1.)*(YL*YL-1.)
0019             SH(7)=-0.5*(XL*XL-1.)*(YL+1.)
0020             SH(8)= 0.5*(XL-1.)*(YL*YL-1.)
0021             RETURN
0022             END
 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = SEREND  , LINECNT =       57
 *STATISTICS*    SOURCE STATEMENTS =       22,PROGRAM SIZE =      868
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
MICHIGAN TERMINAL SYSTEM FORTRAN G(21.8)          FUN1          06-01-82          16:14:59          PAGE P001

 0001                 FUNCTION FUN1(I,M,K)
            C
            C
 0002                 IMPLICIT REAL*8(A-H,O-Z)
 0003                 RI=I
 0004                 RM=M
 0005                 GO TO (100,102,104),K
 0006             100 FUN1=-1.+2.*(RI-1.)/RM
 0007                 RETURN
 0008             102 FUN1=-1.+2.*(RI-1.)*RI/(RM*(RM+1.))
 0009                 RETURN
 0010             104 FUN1=-1.+2.*(RI-1.)*(2.*RM-RI+2.)/(RM*(RM+1.))
 0011                 RETURN
 0012                 END
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = FUN1    , LINECNT =       57
  *STATISTICS*     SOURCE STATEMENTS =       12,PROGRAM SIZE =      634
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
 0001             FUNCTION FTH(TH)                                              935.000
             C                                                                  936.000
             C                                                                  937.000
 0002             IMPLICIT REAL*8(A-H,O-Z)                                      938.000
 0003             COMMON /ADIM/ D,WD,ED,ALD,W,E,AL                              939.000
 0004             Z=W/3.6                                                       940.000
 0005             ZN=Z-(D/2.0)                                                  941.000
 0006             FTH=((((D/2.)+ZN)/DCOS(TH))-(D/2.))/2.+(D/2.)                 942.000
 0007             RETURN                                                        943.000
 0008             END                                                           944.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = FTH     , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =        8,PROGRAM SIZE =       442
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001            SUBROUTINE   POINT(LEB)
0002            IMPLICIT REAL*8(A-H,O-Z)
0003            COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
               1RAN
0004            COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
0005            COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0006            COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
               1              ,SI(4),RI(4),P(20),Q(20)
0007            DIMENSION LEB(20)
          C
          C
0008            RLC=RC+(D/2.0)
0009            RL=RT+(D/2.0)
0010            CA=0.09817477
0011            PI=3.141596535
0012            P(1)=(RL+(RC-RT)*DCOS(CA/6.0))*DSIN(CA/6.0)
0013            Q(1)=(RL+(RC-RT)*DCOS(CA/6.0))*DCOS(CA/6.0)
0014            P(2)=(RL+(RC-RT)*DCOS(CA/2.0))*DSIN(CA/2.)
0015            Q(2)=(RL+(RC-RT)*DCOS(CA/2.0))*DCOS(CA/2.)
0016            P(3)=(RL+(RC-RT)*DCOS(3*CA/2.0))*DSIN(3*CA/2.)
0017            Q(3)=(RL+(RC-RT)*DCOS(3*CA/2.0))*DCOS(3*CA/2.)
0018            P(4)=(RL+(RC-RT)*DCOS(5*CA/2.0))*DSIN(5*CA/2.)
0019            Q(4)=(RL+(RC-RT)*DCOS(5*CA/2.0))*DCOS(5*CA/2.)
0020            P(5)=(RL+(RC-RT)*DCOS(7*CA/2.0))*DSIN(7*CA/2.)
0021            Q(5)=(RL+(RC-RT)*DCOS(7*CA/2.0))*DCOS(7*CA/2.)
0022            P(6)=(RL+(RC-RT)*DCOS(9*CA/2.0))*DSIN(9*CA/2.)
0023            Q(6)=(RL+(RC-RT)*DCOS(9*CA/2.0))*DCOS(9*CA/2.)
0024            P(7)=(RL+(RC-RT)*DCOS(11*CA/2.0))*DSIN(11*CA/2.)
0025            Q(7)=(RL+(RC-RT)*DCOS(11*CA/2.0))*DCOS(11*CA/2.)
0026            P(8)=(RL+(RC-RT)*DCOS(13*CA/2.0))*DSIN(13*CA/2.)
0027            Q(8)=(RL+(RC-RT)*DCOS(13*CA/2.0))*DCOS(13*CA/2.)
0028            P(9)=(RL+(RC-RT)*DCOS(15*CA/2.0))*DSIN(15*CA/2.)
0029            Q(9)=(RL+(RC-RT)*DCOS(15*CA/2.0))*DCOS(15*CA/2.)
0030            P(10)=(RL+(RC-RT)*DCOS(17*CA/2.0))*DSIN(17*CA/2.)
0031            Q(10)=(RL+(RC-RT)*DCOS(17*CA/2.0))*DCOS(17*CA/2.)
0032            P(11)=(RL+(RC-RT)*DCOS(19*CA/2.0))*DSIN(19*CA/2.)
0033            Q(11)=(RL+(RC-RT)*DCOS(19*CA/2.0))*DCOS(19*CA/2.)
0034            P(12)=(RL+(RC-RT)*DCOS(21*CA/2.0))*DSIN(21*CA/2.)
0035            Q(12)=(RL+(RC-RT)*DCOS(21*CA/2.0))*DCOS(21*CA/2.)
0036            P(13)=(RL+(RC-RT)*DCOS(23*CA/2.0))*DSIN(23*CA/2.)
0037            Q(13)=(RL+(RC-RT)*DCOS(23*CA/2.0))*DCOS(23*CA/2.)
0038            P(14)=(RL+(RC-RT)*DCOS(25*CA/2.0))*DSIN(25*CA/2.)
0039            Q(14)=(RL+(RC-RT)*DCOS(25*CA/2.0))*DCOS(25*CA/2.)
0040            P(15)=(RL+(RC-RT)*DCOS(27*CA/2.0))*DSIN(27*CA/2.)
0041            Q(15)=(RL+(RC-RT)*DCOS(27*CA/2.0))*DCOS(27*CA/2.)
0042            P(16)=(RL+(RC-RT)*DCOS(29*CA/2.0))*DSIN(29*CA/2.)
0043            Q(16)=(RL+(RC-RT)*DCOS(29*CA/2.0))*DCOS(29*CA/2.)
0044            P(17)=(RL+(RC-RT)*DCOS(31*CA/2.0))*DSIN(31*CA/2.)
0045            Q(17)=(RL+(RC-RT)*DCOS(31*CA/2.0))*DCOS(31*CA/2.)
0046            P(18)=(RL+(RC-RT)*DSIN(CA/6.0))*DCOS(CA/6.0)
0047            Q(18)=(RL+(RC-RT)*DSIN(CA/6.0))*DSIN(CA/6.0)
0048            I=0
0049          5 I=I+1
0050            IF(I .GT. 8) GO TO 110
0051            RLX=RLC-Y(I)
```

```
0052            IF(RLX .GT. 0.0`    GO TO 5
0053            LEB(1)=I-1
0054            LEB(2)=I-1
0055          6 K=1
0056            LI=0
0057         50 K=K+1
0058            IF(K .GT. 16) GO TO 100
0059            KP=7
0060            IF(K .EQ. 5) KP=19
0061            IF(K .EQ. 9) KP=28
0062            IF(K .EQ. 13) KP=16
           C
0063            M=LEB(K)+KP+LI
0064            LI=0
0065            II=IJK(1,M)
0066            KK=IJK(3,M)
0067            K1=K+1
0068         40 IF(X(II) .EQ. X(KK)) TT=P(K1)-X(KK)
0069            IF(X(II) .EQ. X(KK)) GO TO 45
0070            TT=(-(P(K1)-X(II))*(Y(II)-Y(KK))/(X(II)-X(KK)))-Y(II)+Q(K1)
0071         45 IF(TT .GT. 0.0) GO TO 10
0072            LEB(K+1)=M-1
0073            II=IJK(1,M-1)
0074            KK=IJK(3,M-1)
0075            M=LEB(K+1)
0076            LI=LI+1
0077            GO TO 40
0078         10 LEB(K1)=M
0079            GO TO 50
0080        100 LEB(K+1)=M
0081        110 RETURN
0082            END
 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = POINT    , LINECNT =       57
 *STATISTICS*    SOURCE STATEMENTS =       82,PROGRAM SIZE =     5002
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001             SUBROUTINE  MATRL
0002             IMPLICIT REAL*8(A-H,O-Z)
0003             COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
                1RAN
0004             COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
0005             COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
                1)
           C
           C
0006             IF(NRAN .EQ. 0) NLY=2.0*ANG/DANG+1
0007             V21=E2*V12/E1
0008             DIV=1-V12*V21
0009             Q22=E2/DIV
0010             Q11=E1/DIV
0011             Q12=V12*E2/DIV
0012             Q66=G12
           C
           C   *** COMPUTE INVARIANT PROPERTIES
0013             U1=(3.*Q11+3.*Q22+2.*Q12+4.*Q66)/8.
0014             U2=(Q11-Q22)/2.
0015             U3=(Q11+Q22-2.*Q12-4.*Q66)/8.0
0016             U4=(Q11+Q22+6.*Q12-4.*Q66)/8.0
0017             U5=(Q11+Q22-2.*Q12+4.*Q66)/8.0
           C
           C   ***  COMPUTRE TRANSFORMED REDUCED STIFFNESS PER LAYER *******
           C
0018             DO 100 I=1,NLY
0019             L=I-1
0020             THTA=90.0-(ANG-DANG*L)
0021             IF(NRAN .EQ. 1) THTA=90.0-ANT(I)
0022             IF(THTA .EQ. 180.0) THTA=0.0
0023             DEG=THTA*3.1415926535/180.0
0024             QA(I,1,1)=U1+U2*DCOS(2.*DEG)+U3*DCOS(4.*DEG)
0025             QA(I,1,2)=U4-U3*DCOS(4.*DEG)
0026             QA(I,2,2)=U1-U2*DCOS(2.*DEG)+U3*DCOS(4.*DEG)
0027             QA(I,1,3)=+0.5*U2*DSIN(2.*DEG)+U3*DSIN(4.*DEG)
0028             QA(I,2,3)=+0.5*U2*DSIN(2.*DEG)-U3*DSIN(4.*DEG)
0029             QA(I,3,3)=U5-U3*DCOS(4.*DEG)
0030             QA(I,2,1)=QA(I,1,2)
0031             QA(I,3,1)=QA(I,1,3)
0032             QA(I,3,2)=QA(I,2,3)
           C
           C   ***  COMPUTE ROTATION TRANSFORMATION PER PLY  *****
           C
0033             T(I,1,1)=DCOS(DEG)**2
0034             T(I,1,2)=DSIN(DEG)**2
0035             T(I,1,3)=2*DSIN(DEG)*DCOS(DEG)
0036             T(I,2,1)=T(I,1,2)
0037             T(I,2,2)=T(I,1,1)
0038             T(I,2,3)=-T(I,1,3)
0039             T(I,3,1)=T(I,2,3)/2.0
0040             T(I,3,2)=T(I,1,3)/2.0
0041             T(I,3,3)=T(I,1,1)-T(I,1,2)
           C
           C
```

```
0042         100 CONTINUE                                                   1084.000
           C                                                                1085.000
           C    *** COMPUTE LAMINATE PROPERTIES  *****                      1086.000
           C                                                                1087.000
0043             DO 200 M=1,3                                               1088.000
0044             DO 200 N=1,3                                               1089.000
0045             C(M,N)=0.0                                                 1090.000
0046             DO 150 I=1,NLY                                             1091.000
0047             AZ=THICK(I)                                                1092.000
0048             IF(NRAN .EQ. 0) AZ=1.0                                     1093.000
0049         150 C(M,N)=C(M,N)+QA(I,M,N)*AZ                                 1094.000
0050             IF(NRAN .EQ. 0) GO TO 10                                   1095.000
0051             C(M,N)=C(M,N)/HI                                           1096.000
0052             GO TO 200                                                  1097.000
0053          10 C(M,N)=C(M,N)/NLY                                          1098.000
0054         200 CONTINUE                                                   1099.000
0055             RETURN                                                     1100.000
0056             END                                                        1101.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = MATRL    , LINECNT =       57
  *STATISTICS*     SOURCE STATEMENTS =       56,PROGRAM SIZE =      2044
  *STATISTICS*  NO DIAGNOSTICS GENERATED
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```
 0001             SUBROUTINE STIFF(MM,A)                                           1102.000
 0002             IMPLICIT REAL*8(A-H,O-Z)                                         1103.000
            C                                                                      1104.000
 0003             COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)                    1105.000
 0004             COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND              1106.000
 0005             COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)                   1107.000
                 1                  ,SI(4),RI(4),P(20),Q(20)                       1108.000
 0006             COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3  1109.000
                 1)                                                                1109.000
            C                                                                      1110.000
 0007             DIMENSION A(8,8)                                                 1111.000
 0008             DO 10 I=1,8                                                      1112.000
 0009             DO 10 J=1,8                                                      1114.000
 0010             A(I,J)=0.0                                                       1115.000
 0011          10 CONTINUE                                                         1116.000
            C                                                                      1117.000
 0012             DO 100 N=1,LINT                                                  1118.000
 0013             R=RG(N)                                                          1119.000
 0014             DO 100 M=1,LINT                                                  1120.000
 0015             S=SG(M)                                                          1121.000
 0016             CALL SHAPEF(MM,XSJ,R,S)                                          1122.000
 0017             DV=XSJ*WG(N)*WG(M)                                               1123.000
            C                                                                      1124.000
            C   *** FOR EACH J NODE COMPUTE CB=C*B  ***                            1125.000
            C                                                                      1126.000
 0018             DO 100 J=1,NP                                                    1127.000
 0019             CB11=C(1,1)*SHP(1,J)*DV+C(1,3)*SHP(2,J)*DV                       1128.000
 0020             CB12=C(1,2)*SHP(2,J)*DV+C(1,3)*SHP(1,J)*DV                       1129.000
 0021             CB21=C(1,2)*SHP(1,J)*DV+C(2,3)*SHP(2,J)*DV                       1130.000
 0022             CB22=C(2,2)*SHP(2,J)*DV+C(2,3)*SHP(1,J)*DV                       1131.000
 0023             CB31=C(1,3)*SHP(1,J)*DV+C(3,3)*SHP(2,J)*DV                       1132.000
 0024             CB32=C(2,3)*SHP(2,J)*DV+C(3,3)*SHP(1,J)*DV                       1133.000
            C                                                                      1134.000
            C   *** FOR EACH I NODE COMPUTE S=BT*CB  ****                          1135.000
            C                                                                      1136.000
 0025             DO 100 I=1,J                                                     1137.000
 0026             I1=2*I-1                                                         1138.000
 0027             J1=2*J-1                                                         1139.000
 0028             I2=2*I                                                           1140.000
 0029             J2=2*J                                                           1141.000
            C                                                                      1142.000
 0030             A(I1,J1)=A(I1,J1)+SHP(1,I)*CB11+SHP(2,I)*CB31                    1143.000
 0031             A(I1,J2)=A(I1,J2)+SHP(1,I)*CB12+SHP(2,I)*CB32                    1144.000
 0032             A(I2,J1)=A(I2,J1)+SHP(2,I)*CB21+SHP(1,I)*CB31                    1145.000
 0033             A(I2,J2)=A(I2,J2)+SHP(2,I)*CB22+SHP(1,I)*CB32                    1146.000
 0034         100 CONTINUE                                                         1147.000
            C                                                                      1148.000
            C   *** COMPUTE LOWER TRIANGULAR PART BY SYMMETRY  ***                 1149.000
            C                                                                      1150.000
 0035             NL=NP*2                                                          1151.000
 0036             DO 200 I=1,NL                                                    1152.000
 0037             DO 200 J=1,I                                                     1153.000
 0038         200 A(I,J)=A(J,I)                                                    1154.000
            C                                                                      1155.000
 0039             RETURN                                                           1156.000
```

```
0040          END                                                                    1157.000
 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = STIFF    , LINECNT =       57
 *STATISTICS*    SOURCE STATEMENTS =       40,PROGRAM SIZE =      1516
 *STATISTICS*  NO DIAGNOSTICS GENERATED
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```
MICHIGAN TERMINAL SYSTEM FORTRAN G(21.8)              CFAIL          06-01-82         16:15:07         PAGE P001

  0001               SUBROUTINE CFAIL (KT,MT,FAL)                                       1158.000
  0002               IMPLICIT REAL*8(A-H,O-Z)                                           1159.000
  0003               COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N   1160.000
                    1RAN                                                                1160.000
  0004               COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3   1161.000
                    1)                                                                  1161.000
               C                                                                        1162.000
               C                                                                        1163.000
               C   *** FAILURE CRITERIAN  ******                                        1164.000
               C                                                                        1165.000
  0005               FAL=0.0                                                            1166.000
  0006               DO 100 K=1,NLY                                                     1167.000
  0007               DO 100 M=1,18                                                      1168.000
               C                                                                        1169.000
  0008               FL=(STRSS(K,M,1)/XX)**2+(STRSS(K,M,3)/SS)**2                       1170.000
  0009               IF (FL .LT. FAL) GO TO 100                                         1171.000
  0010               FAL=FL                                                             1172.000
  0011               KT=K                                                               1173.000
  0012               MT=M                                                               1174.000
  0013               IF(FAL .GT. 1.0) GO TO 200                                         1175.000
  0014           100 CONTINUE                                                           1176.000
               C                                                                        1177.000
  0015           200 RETURN                                                             1178.000
  0016               END                                                                1179.000
   *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
   *OPTIONS IN EFFECT*  NAME = CFAIL    , LINECNT =       57
   *STATISTICS*    SOURCE STATEMENTS =       16,PROGRAM SIZE =       584
   *STATISTICS*  NO DIAGNOSTICS GENERATED
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```
0001            SUBROUTINE  STRESS(DIS)                                             1180.000
0002            IMPLICIT REAL*8(A-H,O-Z)                                            1181.000
          C                                                                         1182.000
0003            COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N  1183.000
               1RAN                                                                 1183.000
0004            COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)                       1184.000
0005            COMMON /ALEM/ NODXY,NELX,IJK(4,400)                                 1185.000
0006            COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)                      1186.000
               1                ,SI(4),RI(4),P(20),Q(20)                            1187.000
0007            COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3  1188.000
               1)                                                                   1188.000
0008             COMMON /ALOA/ XLC(20),YLC(20)                                      1189.000
          C                                                                         1190.000
          C                                                                         1191.000
          C                                                                         1192.000
0009            DIMENSION HP(3,4),LEB(20),XI(4),YI(4),DIS(2,400),STRAS(11,3)        1193.000
0010            DO 10 I=1,18                                                        1194.000
0011            DO 10 J=1,3                                                         1195.000
0012            STRA(I,J)=0.0                                                       1196.000
0013         10 CONTINUE                                                            1197.000
          C                                                                         1198.000
0014            DO 13 I=1,20                                                        1199.000
0015            XLC(I)=0.0                                                          1200.000
0016         13 YLC(I)=0.0                                                          1201.000
          C                                                                         1202.000
0017            CALL POINT(LEB)                                                     1203.000
          C                                                                         1204.000
0018            DO 200 M=1,18                                                       1205.000
0019            II=LEB(M)                                                           1206.000
0020            II1=IJK(1,II)                                                       1207.000
0021            II2=IJK(2,II)                                                       1208.000
0022            II3=IJK(3,II)                                                       1209.000
0023            II4=IJK(4,II)                                                       1210.000
          C                                                                         1211.000
0024            XI(1)=X(II1)                                                        1212.000
0025            XI(2)=X(II2)                                                        1213.000
0026            XI(3)=X(II3)                                                        1214.000
0027            XI(4)=X(II4)                                                        1215.000
          C                                                                         1216.000
0028            YI(1)=Y(II1)                                                        1217.000
0029            YI(2)=Y(II2)                                                        1218.000
0030            YI(3)=Y(II3)                                                        1219.000
0031            YI(4)=Y(II4)                                                        1220.000
          C                                                                         1221.000
0032            AP=P(M)                                                             1222.000
0033            AQ=Q(M)                                                             1223.000
          C                                                                         1224.000
0034            CALL NEWTON (II,AP,AQ,R,S)                                          1225.000
          C                                                                         1226.000
0035            CALL SHAPEF(II,XSJ,R,S)                                             1227.000
          C                                                                         1228.000
0036            DO 100 N=1,4                                                        1229.000
0037            NN=IJK(N,II)                                                        1230.000
0038            STRA(M,1)=STRA(M,1)+SHP(1,N)*DIS(1,NN)                              1231.000
0039            STRA(M,2)=STRA(M,2)+SHP(2,N)*DIS(2,NN)                              1232.000
```

```
0040             STRA(M,3)=STRA(M,3)+SHP(1,N)*DIS(2,NN)+SHP(2,N)*DIS(1,NN)
           C
0041         100 CONTINUE
           C
0042             DO 50 K=1,NLY
0043             DO 40 L=1,3
0044             STRSS(K,M,L)=0.0
0045             DO 30 I=1,3
0046             DO 20 J=1,3
           C
0047             STRSS(K,M,L)=STRSS(K,M,L)+T(K,L,I)*QA(K,I,J)*STRA(M,J)
           C
0048          20 CONTINUE
0049          30 CONTINUE
0050          40 CONTINUE
0051          50 CONTINUE
           C
0052         200 CONTINUE
0053         250 DO 11 I=1,11
0054             DO 11 J=1,3
0055          11 STRAS(I,J)=0.0
           C
0056             MX=148
0057             MY=154
0058             MT=0
0059             DO 400 IN=MX,MY
0060             MT=MT+1
0061             II1=IJK(1,IN)
0062             II2=IJK(2,IN)
0063             II3=IJK(3,IN)
0064             II4=IJK(4,IN)
           C
0065             XI(1)=X(II1)
0066             XI(2)=X(II2)
0067             XI(3)=X(II3)
0068             XI(4)=X(II4)
           C
0069             YI 1)=Y(II1)
0070             YI(2)=Y(II2)
0071             YI(3)=Y(II3)
0072             YI(4)=Y(II4)
           C
0073             AP=RG(1)
0074             AQ=SG(2)
           C
0075             CALL SHAPEF(IN,XSJ,AP,AQ)
           C
0076             DO 500 NT=1,4
0077             NN=IJK(NT,IN)
           C
0078             XLC(MT)=XLC(MT)+SHP(3,NT)*X(NN)
0079             YLC(MT)=YLC(MT)+SHP(3,NT)*Y(NN)
           C
           C
0080             STRAS(MT,1)=STRAS(MT,1)+SHP(1,NT)*DIS(1,NN)
```

```
0081              STRAS(MT,2)=STRAS(MT,2)+SHP(2,NT)*DIS(2,NN)
0082              STRAS(MT,3)=STRAS(MT,3)+SHP(1,NT)*DIS(2,NN)+SHP(2,NT)*DIS(1,NN)
          C
0083          500 CONTINUE
          C
0084              DO 520 J=1,3
0085          520 ST(MT,J)=0.0
0086              DO 550 J=1,3
0087              ST(MT,1)=ST(MT,1)+C(1,J)*STRAS(MT,J)
0088              ST(MT,2)=ST(MT,2)+C(2,J)*STRAS(MT,J)
0089              ST(MT,3)=ST(MT,3)+C(3,J)*STRAS(MT,J)
          C
0090          550 CONTINUE
          C
          C
0091          400 CONTINUE
          C
          C
0092              MX=164
0093              MY=166
0094              MT=7
0095              DO 401 IN=MX,MY
0096              MT=MT+1
0097              II1=IJK(1,IN)
0098              II2=IJK(2,IN)
0099              II3=IJK(3,IN)
0100              II4=IJK(4,IN)
          C
0101              XI(1)=X(II1)
0102              XI(2)=X(II2)
0103              XI(3)=X(II3)
0104              XI(4)=X(II4)
          C
0105              YI(1)=Y(II1)
0106              YI(2)=Y(II2)
0107              YI(3)=Y(II3)
0108              YI(4)=Y(II4)
          C
0109              AP=RG(1)
0110              AQ=SG(2)
          C
0111              CALL SHAPEF(IN,XSJ,AP,AQ)
          C
0112              DO 501 NT=1,4
0113              NN=IJK(NT,IN)
          C
0114              XLC(MT)=XLC(MT)+SHP(3,NT)*X(NN)
0115              YLC(MT)=YLC(MT)+SHP(3,NT)*Y(NN)
          C
          C
0116              STRAS(MT,1)=STRAS(MT,1)+SHP(1,NT)*DIS(1,NN)
0117              STRAS(MT,2)=STRAS(MT,2)+SHP(2,NT)*DIS(2,NN)
0118              STRAS(MT,3)=STRAS(MT,3)+SHP(1,NT)*DIS(2,NN)+SHP(2,NT)*DIS(1,NN)
          C
0119          501 CONTINUE
```

```
                C                                                                   1343.000
  0120                DO 521 J=1,3                                                  1344.000
  0121            521 ST(MT,J)=0.0                                                  1345.000
  0122                DO 551 J=1,3                                                  1346.000
  0123                ST(MT,1)=ST(MT,1)+C(1,J)*STRAS(MT,J)                          1347.000
  0124                ST(MT,2)=ST(MT,2)+C(2,J)*STRAS(MT,J)                          1348.000
  0125                ST(MT,3)=ST(MT,3)+C(3,J)*STRAS(MT,J)                          1349.000
                C                                                                   1350.000
  0126            551 CONTINUE                                                      1351.000
                C                                                                   1352.000
                C                                                                   1353.000
  0127            401 CONTINUE                                                      1354.000
                CC     DO 666 I=1,10                                                1355.000
                C     WRITE(6,677) I,(STRAS(I,J),J=1,3)                             1356.000
                C 677 FORMAT(/,5X,'POINT=',I5,5X,'STRA11=',E15.8,5X,'STRA22=',E15.8, 1357.000
                C    15X,'STRA12=',E15.8,/)                                         1358.000
                C 666 CONTINUE                                                      1359.000
                C                                                                   1360.000
  0128            300 RETURN                                                        1361.000
  0129                END                                                           1362.000
   *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
   *OPTIONS IN EFFECT*  NAME = STRESS   , LINECNT =       57
   *STATISTICS*    SOURCE STATEMENTS =      129,PROGRAM SIZE =     4434
   *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001          SUBROUTINE  OUTPUT(KT,MT,FAL)
0002          IMPLICIT REAL*8(A-H,O-Z)
        C
0003          COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
             1RAN
0004          COMMON /AMAT/ E1,E2,V12,G12,C(3,3),QA(40,3,3)
0005          COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
0006          COMMON / AFORE/ PF,DP,F(706)
0007          COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0008          COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0009          COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
             1              ,SI(4),RI(4),P(20),Q(20)
0010          COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
             1)
0011           COMMON /ALOA/ XLC(20),YLC(20)
        C
        C
        C
0012          HIH=2.0*HI
        C
0013          SXS1=STRSS(KT,MT,1)*DSQRT(1.0/FAL)
0014          SXS2=STRSS(KT,MT,2)*DSQRT(1.0/FAL)
0015          SXS3=STRSS(KT,MT,3)*DSQRT(1.0/FAL)
        C
0016          TP=2000.0*HI
        C
        C
0017          WRITE(6,15)
0018       15 FORMAT(///,'----------------------------------------------------------
             1-------------------------------------------------------',/)
0019          IF(ITEST .EQ. 0) WRITE(6,7)
0020        7 FORMAT(/,10X,'****************** THE STRENGTH PREDICTION OF
             1LAMINATE CONTAINING A OPEN HOLE     ********************'/)
        C
0021          IF(ITEST .EQ. 1) WRITE(6,100)
0022      100 FORMAT(/,10X,'**************   THE STRENGTH PREDICTION OF
             1FASTENED COMPOSITE JOINTS   **********'/)
0023          WRITE(6,16)
0024       16 FORMAT(/,'----------------------------------------------------------
             1---------------------------------------------------',/)
0025          WRITE (6,901)
0026          WRITE (6,311) E1,V12,G12
0027          WRITE (6,888)
0028      888 FORMAT(//,20X,'MATERIAL PROPERTIES')
0029          WRITE(6,889) XX,SS,RT,RC
0030      889 FORMAT(/20X,'X(PLY T-STRENGTH)=',F15.3,5X,'SC(LAMINATE S-STRENGTH)
             1=',
             1F15.3,//,20X,'RT(CHAR. - TEN.)=',F7.4,5X,'RC(CHAR. - COMP.)=',
             2F7.4,/)
0031          WRITE (6,902)
0032          IF(NRAN .EQ. 0) WRITE(6,213) ANG,DANG
0033          IF(NRAN .EQ. 1) WRITE(6,903) (I,ANT(I),THICK(I),I=1,NLY)
0034          WRITE(6,904)
0035          WRITE (6,400) D,WD,ED,ALD,HIH
0036          WRITE(6,905)
```

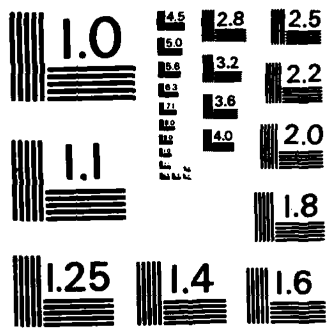
AD-A121 407 STRENGTH OF MECHANICALLY FASTENED COMPOSITE JOINTS(U) 2/2
MICHIGAN UNIV ANN ARBOR DEPT OF MECHANICAL ENGINEERING
AND APPLIED MECHANICS F CHANG ET AL. JUL 82

UNCLASSIFIED AFWAL-TR-82-4095 F33615-81-C-5050 F/G 13/5 NL

END

FILMED

DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

```
              C
0037            901 FORMAT (//,20X,'MATERIAL PROPERTIES OF SINGLE PLY')
0038            311 FORMAT (/,20X,'E1=',F15.3,5X,'V12=',F10.3,
                   15X,'G12=',F15.3/)
0039            213 FORMAT (/,20X,'**  THE MAXIMUM ORIENTATION ANGLE=',
                   11X,F6.2,5X,'** THE INCREMENTAL ORIENTATION ANGLE=',
                   21X,F6.2/)
0040            400 FORMAT (//,20X,'DIAMETER=',F10.4,5X,'W/D=',F10.4,/,20X,'E/D=',
                   1F10.4,5X,'L/D=',F10.4/,20X,'THICKNESS=',F10.5//)
0041            902 FORMAT (//,20X,'PLY ORIENTATION ',10X,'PLY THICKNESS')
0042            903 FORMAT (/,20X,'PLY',I3,'=',F7.3,11X,F8.5,5X)
0043            904 FORMAT (///,20X,'GEOMETRY')
0044            905 FORMAT('-----------------------------------------------------------
                   1--------------------------------------------------------',///)
              C
0045                WRITE(6,101) PF
0046            101 FORMAT(/,10X,'******   THE MAXIMUM LOAD=',F10.3,2X,'    *******',1
                   1X)
0047                IF(MT .LE. 5) WRITE(6,91)
0048                IF(MT .LE. 11 .AND. MT .GE. 8) WRITE(6,92)
0049                IF(MT .GE. 14 .AND. MT .LE. 18) WRITE(6,93)
0050                IF(MT .LE. 7 .AND. MT .GE. 6) WRITE(6,94)
0051                IF(MT .GE. 12 .AND. MT .LE. 13 ) WRITE(6,95)
0052                WRITE(6,102)
0053            102 FORMAT(/,3X,'FAILURE LAYER',3X,'FAILURE POINT',
                   13X,'FAILURE POSITION',3X,'STRESS11',3X,'STRESS22',3X,'STRESS12',/)
0054                WRITE(6,103) KT,MT,P(MT),Q(MT),SXS1,SXS2,SXS3
0055            103 FORMAT (/,7X,I3,12X,I3,7X,'(',1X,F5.3,2X,F5.3,')',3X,E10.4,
                   13X,E10.4,3X,E10.4,//)
              C
0056                WRITE(6,96)
0057                WRITE(6,97) TP
0058                WRITE(6,98) FAL
0059                WRITE(6,99)
              C
              C
0060             91 FORMAT(/,10X,'******','   THE FAILURE MODE = BEARING MODE',//)
0061             92 FORMAT(/,10X,'******','   THE FAILURE MODE = SHEAROUT MODE',//)
0062             93 FORMAT(/,10X,'******','   THE FAILURE MODE = TENSION MODE',//)
0063             94 FORMAT(/,10X,'******','   THE FAILURE MODE = BEARING AND SHEAROUT
                   1MODE',//)
0064             95 FORMAT(/,10X,'******','   THE FAILURE MODE = TENSION AND SHEAROUT
                   1MODE',//)
0065             96 FORMAT(/,'-----------------------------------------------------------
                   1-------------------------------------------------------',/)
0066             97 FORMAT(/,10X,'******','   THE INITIAL LOAD=',F15.5,/)
0067             98 FORMAT(/,10X,'******','   THE FAILURE INDICTOR =',F7.4,//)
0068             99 FORMAT(///,'*****************   THE STRESS DISTRIBUTIONS DUE TO
                   1THE INITIAL LOAD ON THE CHARACTERISTIC CURVE FOR EACH PLY   *',//)
0069                DO 200 K=1,NLY
0070                WRITE(6,104)
0071            104 FORMAT(/,3X,'LAYER',7X,'NO.',5X,'X1',8X,'X2',8X,'TXX',10X,'TYY',10
                   1X,'TXY',/)
0072                DO 150 N=1,18
0073                WRITE(6,105) K,N,P(N),Q(N),STRSS(K,N,1),STRSS(K,N,2),STRSS(K,N,3)
```

```
0074         105 FORMAT(/,5X,I3,5X,I3,5X,F6.3,3X,F6.3,6X,E10.4,5X,E10.4,5X,E10.4,/)
0075         150 CONTINUE
           C
0076         200 CONTINUE
                 WRITE(6,17)
0077
0078          17 FORMAT (///,'********************   THE STRESS DISTRIBUTIONS
                1ACROSS THE LIGAMENT OF THE PLATE   ************************',/)
           C
0079         300 DO 301 I=1,10
0080             WRITE(6,305) I,XLC(I),YLC(I),(ST(I,J),J=1,3)
0081         305 FORMAT(/,5X,'POINT=',I5,5X,'X1=',F7.4,5X,'X2=',F7.4,5X,'T11=',
                1E15.8,5X,'T22=',E15.8,5X,'T12=',E15.8,/)
0082         301 CONTINUE
           C
0083             GO TO 500
0084         500 RETURN
0085             END
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = OUTPUT   , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       85,PROGRAM SIZE =     4414
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001            SUBROUTINE  FORMK
0002            IMPLICIT REAL*8 (A-H,O-Z)
0003            COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0004            COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0005            COMMON /AMAX/ SK(706,90),R1(706)
          C
          C
          C
0006            DIMENSION A(8,8)
          C
          C
0007            DO 300 N=1,NEQ
0008            DO 300 M=1,NBAND
0009        300 SK(N,M)=0.0
          C
          C
0010            DO 400 N=1,NELX
          C
0011            CALL STIFF(N,A)
          C
0012            DO 350 JJ=1,NP
0013            NROWB=(IJK(JJ,N)-1)*2
0014            DO 350 J=1,2
0015            NROWB=NROWB+1
0016            I=(JJ-1)*2+J
0017            DO 330 KK=1,NP
0018            NCOLB=(IJK(KK,N)-1)*2
0019            DO 320 K=1,2
0020            L=(KK-1)*2+K
0021            NCOL=NCOLB+K+1-NROWB
          C
0022            IF(NCOL) 320,320,310
          C
0023        310 SK(NROWB,[illegible])=SK(NROWB,NCOL)+A(I,L)
          C
0024        320 CONTIN
0025        330 CONTIN
0026        350 CONTI[illegible]
0027        400 CONTIN[illegible]
          C
0028            RETURN
0029            END
 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = FORMK    , LINECNT =       57
 *STATISTICS*    SOURCE STATEMENTS =       29,PROGRAM SIZE =     1428
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001             SUBROUTINE BOUND
0002             IMPLICIT REAL*8 (A-H,O-Z)
0003             COMMON /ACONT/ ANG,DANG,NLY,ITEST,XX,SS,RT,RC,HI,ANT(9),THICK(9),N
                1RAN
0004             COMMON /ADIM/ D,WD,ED,ALD,W,E,AL
0005             COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0006             COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0007             COMMON /AMAX/ SK(706,90),R1(706)
           C
0008             NB=37
0009             IF(ITEST .EQ. 1) NB=33
           C
0010             DO 500 N=1,NB
0011             NX=10
0012             I=NBC(N)
0013             NROWB=(I-1)*2
           C
           C
           C   **** EXAMINE EACH DEGREE OF FREEDOM  *****
           C
0014             DO 490 M=1,2
0015             NROWB=NROWB+1
0016             ICON=NFIX(N)/NX
           C
0017             IF(ICON) 450,450,420
           C
0018         420 SK(NROWB,1)=1.0
0019             DO 430 J=2,NBAND
0020             SK(NROWB,J)=0.0
0021             NR=NROWB+1-J
           C
0022             IF(NR) 430,430,425
           C
0023         425 SK(NR,J)=0.0
0024         430 CONTINUE
           C
0025             NFIX(N)=NFIX(N)-NX*ICON
0026         450 NX=NX/10
0027         490 CONTINUE
0028         500 CONTINUE
           C
0029             RETURN
0030             END
 *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
 *OPTIONS IN EFFECT*  NAME = BOUND    , LINECNT =       57
 *STATISTICS*    SOURCE STATEMENTS =        30,PROGRAM SIZE =       740
 *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001             SUBROUTINE SOLVE
0002             IMPLICIT REAL*8 (A-H,O-Z)
0003             COMMON / AFORE/ PF,DP,F(706)
0004             COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0005             COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0006             COMMON /AMAX/ SK(706,90),R1(706)
           C
0007             DO 100 I=1,NEQ
0008             R1(I)=F(I)
0009         100 CONTINUE
           C
0010             DO 300 N=1,NEQ
0011             I=N
0012             DO 290 L=2,NBAND
0013             I=I+1
           C
0014             IF(I-NEQ) 230,230,290
           C
0015         230 IF(SK(N,L)) 240,290,240
           C
0016         240 C1=SK(N,L)/SK(N,1)
0017             J=0
0018             DO 270 K=L,NBAND
0019             J=J+1
           C
0020             IF(SK(N,K)) 260,270,260
           C
0021         260 SK(I,J)=SK(I,J)-C1*SK(N,K)
0022         270 CONTINUE
           C
0023         280 SK(N,L)=C1
0024             R1(I)=R1(I)-C1*R1(N)
           C
0025         290 CONTINUE
           C
0026         300 R1(N)=R1(N)/SK(N,1)
           C
           C   **** BACK SUSTITUTION   ******
           C
           C
0027             N=NEQ
0028         350 N=N-1
0029             IF(N) 500,500,360
           C
0030         360 L=N
0031             DO 400 K=2,NBAND
0032             L=L+1
           C
0033             IF(SK(N,K)) 370,400,370
           C
0034         370 R1(N)=R1(N)-SK(N,K)*R1(L)
           C
0035         400 CONTINUE
0036             GO TO 350
```

```
 0037         500 RETURN                                                          1622.000
 0038             END                                                             1623.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = SOLVE    , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       38,PROGRAM SIZE =      1068
  *STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001            SUBROUTINE   SHAPEF(MM,XSJ,R,S)
0002            IMPLICIT REAL*8 (A-H,O-Z)
0003            COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0004            COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0005            COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
               1              ,SI(4),RI(4),P(20),Q(20)
0006            COMMON /AFUN/ SHP(3,4),T(40,3,3),STRSS(40,18,3),STRA(18,3),ST(11,3
               1)
          C
          C
0007            DIMENSION XS(2,2)
          C
          C
          C   **** COMPUTE SHAPE FUNCTION AND DEVIVATIVE IN LOCAL COORD. ****
          C
0008            DO 100 I=1,NP
          C
0009            SHP(3,I)=0.25*(1.0+R*RI(I))*(1.0+S*SI(I))
0010            SHP(1,I)=0.25*RI(I)*(1.0+S*SI(I))
0011            SHP(2,I)=0.25*SI(I)*(1.0+R*RI(I))
          C
0012        100   CONTINUE
          C
          C
          C   **** COMPUTE JOCOBIAN MATRIX XS: DX/DR   *****
          C
0013            DO 200 I=1,2
0014            DO 200 J=1,2
0015        200 XS(I,J)=0.0
          C
          C
0016            DO 210 J=1,2
0017            DO 210 K=1,NP
0018            NN=IJK(K,MM)
0019            XS(1,J)=XS(1,J)+X(NN)*SHP(J,K)
0020            XS(2,J)=XS(2,J)+Y(NN)*SHP(J,K)
          C
0021        210 CONTINUE
          C
          C
          C   *** COMPUTE DEETERMINENT OF JOCOBIAN MATRIX    *****
          C
0022            XSJ=XS(1,1)*XS(2,2)-XS(1,2)*XS(2,1)
          C
          C   **** TRANSFER NATURAL DERIVATIVE TO X,Y DERIVATIVE  *****
          C
0023            DO 300 I=1,4
          C
0024            TEMP=(XS(2,2)*SHP(1,I)-XS(2,1)*SHP(2,I))/XSJ
0025            SHP(2,I)=(-XS(1,2)*SHP(1,I)+XS(1,1)*SHP(2,I))/XSJ
          C
0026        300 SHP(1,I)=TEMP
          C
0027            RETURN
0028            END
```

```
*OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
*OPTIONS IN EFFECT*  NAME = SHAPEF   , LINECNT =       57
*STATISTICS*    SOURCE STATEMENTS =        28,PROGRAM SIZE =      1028
*STATISTICS*  NO DIAGNOSTICS GENERATED
```

```
0001            SUBROUTINE NEWTON(IN,AP,AQ,R,S)
0002            IMPLICIT REAL*8 (A-H,O-Z)
0003            COMMON /ACOOR/ X(400),Y(400),RG(2),SG(2),WG(2)
               1              ,SI(4),RI(4),P(20),Q(20)
0004            COMMON /ANOD/ NX,NP,NEQ,NBC(37),NFIX(37),LINT,NBAND
0005            COMMON /ALEM/ NODXY,NELX,IJK(4,400)
0006            DIMENSION RP(2)
0007            F(R,S)=A3*R*S+A2*S+A1*R+AO-AP
0008            G(R,S)=B3*R*S+B2*S+B1*R+BO-AQ
          C
0009            FR(R,S)=A3*S+A1
0010            FS(R,S)=A3*R+A2
          C
0011            GR(R,S)=B3*S+B1
0012            GS(R,S)=B3*R+B2
          C
0013            AO=0.0
0014            A1=0.0
0015            A2=0.0
0016            A3=0.0
          C
0017            BO=0.0
0018            B1=0.0
0019            B2=0.0
0020            B3=0.0
          C
          C
0021            DO 100 I=1,4
0022            N=IJK(I,IN)
          C
0023            AO=AO+0.25*X(N)
0024            A1=A1+0.25*X(N)*RI(I)
0025            A2=A2+0.25*X(N)*SI(I)
0026            A3=A3+0.25*X(N)*RI(I)*SI(I)
          C
0027            BO=BO+0.25*Y(N)
0028            B1=B1+0.25*Y(N)*RI(I)
0029            B2=B2+0.25*Y(N)*SI(I)
0030            B3=B3+0.25*Y(N)*RI(I)*SI(I)
          C
0031        100 CONTINUE
          C
          C
0032            R=0.0
0033            S=0.0
          C
0034            AA=F(R,S)
0035            BB=G(R,S)
          C
          C     WRITE(6,3) R,S,AA,BB
          C
0036            DO 10 I=1,20
0037            XJ=FR(R,S)*GS(R,S)-FS(R,S)*GR(R,S)
0038            IF(XJ .EQ. 0.0)  WRITE(6,6)
0039            IF(XJ .EQ. 0.0) GO TO 150
```

```
              C                                                              1733.000
 0040               DELTAX=(-F(R,S)*GS(R,S)+G(R,S)*FS(R,S))/XJ              1734.000
 0041               DELTAY=(-G(R,S)*FR(R,S)+F(R,S)*GR(R,S))/XJ              1735.000
              C                                                              1736.000
 0042               R=R+DELTAX                                               1737.000
 0043               S=S+DELTAY                                               1738.000
              C                                                              1739.000
 0044               AA=F(R,S)                                                1740.000
 0045               BB=G(R,S)                                                1741.000
              C                                                              1742.000
              C     WRITE(6,4) I,R,S,AA,BB                                   1743.000
 0046               IF(DABS(DELTAX) .LT. 1.E-7 .AND. DABS(DELTAY) .LT. 1.E-7) 1744.000
                   1GO TO 150                                                1745.000
              C                                                              1746.000
 0047            10 CONTINUE                                                 1747.000
 0048               WRITE(6,5)                                               1748.000
              C   3 FORMAT(//,10X,'R',15X,'S',15X,'F(R,S)',15X,'G(R,S)',/,   1749.000
              C    15X,F10.7,5X,F10.7,5X,F12.7,5X,F12.7/)                    1750.000
              C   4 FORMAT(/,'I=',I3,F10.7,5X,F10.7,5X,F12.7,5X,F12.7/)      1751.000
 0049             5 FORMAT(//,'FAIL TO CONVERGE IN 20 ITERATION ',/)         1752.000
 0050             6 FORMAT(//,'***  JOCOBIAN=0.0  ***',/)                    1753.000
              C                                                              1754.000
 0051           150 RETURN                                                   1755.000
 0052               END                                                      1756.000
  *OPTIONS IN EFFECT*  ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,NOMAP
  *OPTIONS IN EFFECT*  NAME = NEWTON   , LINECNT =       57
  *STATISTICS*    SOURCE STATEMENTS =       52,PROGRAM SIZE =     2348
  *STATISTICS*  NO DIAGNOSTICS GENERATED

NO STATEMENTS FLAGGED IN THE ABOVE COMPILATIONS.
```

```
C    ****************************************************************
C
C                             TEST SAMPLE
C
C    *****************************************************************
C
C          CONSIDER A SINGLE FASTENER COMPOSITE JOINT MADE OF
C       T300/SP286 GRAPHITE EPOXY LAMINATE.  THE MATERIAL PRO-
C       PERITIES ARE GIVEN AS E1=18.70 E06 PSI, G12=0.719 E06
C       PSI, V12=0.30, X(PLY TENSILE STRENGTH)=0.15 E 06 PSI,
C       SS(LAMINATE SHEAR STRENGTH--CROSS  PLY)=0.018 E06 PSI.
C       QUASI-ISOTROPIC LAMINATE IS USED IN THIS CALCULATIONS.
C
C          THE GEOMETRY OF THE JOINT ARE DESCRIBED AS BELOW:
C                      D(DIAMETER)=0.25 IN
C                      W/D(WIDTH RATIO)=6.0
C                      E/D(EDAGE RATIO)=3.0
C                      L/D(LENGTH RATIO)=14.0
C                      H(THICKNESS)=0.021 IN
C          THE CHARACTERISTIC LENGTH FOR TENSION IS EQUAL TO
C      0.043 IN AND FOR COMPRESSION IS 0.120 IN.
C
C
C
C_________________________________________________________________
C
C          THE FORMAT OF INPUT DATA IS GIVEN IN THE FOLLOWING:
C
C          (SEE INPUT INSTRUCTIONS IN PROGRAM )
CC
C
    1
    1
          0.0            1.0
  18700000.0            0.30        719000.0
      0.2500           6.000           3.000            14.0
     150000.         18000.0           0.043          0.1200
       0.021
    4
         0.0         0.00525            45.0         0.00525
       -45.0         0.00525            90.0         0.00525
```

Output

```
------------------------------------------------------------------------------------------
     ***************   THE STRENGTH PREDICTION OF      FASTENED COMPOSITE JOINTS   ***********

------------------------------------------------------------------------------------------
               MATERIAL PROPERTIES OF SINGLE PLY
               E1=   18700000.000     V12=      0.300     G12=      719000.000
               MATERIAL PROPERTIES
               X(PLY T-STRENGTH)=        150000.000    SC(LAMINATE S-STRENGTH)=      18000.000
               RT(CHAR. - TEN.)= 0.0430      RC(CHAR. - COMP.)= 0.1200
               PLY ORIENTATION              PLY THICKNESS
               PLY   1=  0.0                0.00525
               PLY   2= 45.000              0.00525
               PLY   3=-45.000              0.00525
               PLY   4= 90.000              0.00525
               GEOMETRY
               DIAMETER=     0.2500     W/D=     6.0000
               E/D=     3.0000     L/D=    14.0000
               THICKNESS=    0.04200

------------------------------------------------------------------------------------------
      ******   THE MAXIMUM LOAD=  1161.505       *******
      ******   THE FAILURE MODE = BEARING AND SHEAROUT MODE
 FAILURE LAYER   FAILURE POINT   FAILURE POSITION   STRESS11    STRESS22    STRESS12
       2               6        ( 0.102  0.215)   -.1448E+06  0.1820E+00  -.4687E+04

------------------------------------------------------------------------------------------
      ******   THE INITIAL LOAD=          42.00000
      ******   THE FAILURE INDICTOR = 0.0013
*****************  THE STRESS DISTRIBUTIONS DUE TO   THE INITIAL LOAD ON THE CHARACTERISTIC CURVE FOR EACH PLY   *
 LAYER       NO.     X1       X2         TXX          TYY          TXY
     1        1    0.004    0.245    -.4941E+04   0.8567E-02   0.3624E+02
     1        2    0.012    0.245    -.4906E+04   0.8634E-02   0.4183E+02
     1        3    0.036    0.242    -.4525E+04   0.7968E-02   0.1229E+03
     1        4    0.059    0.235    -.3780E+04   0.6694E-02   0.1953E+03
     1        5    0.081    0.226    -.2723E+04   0.4975E-02   0.2530E+03
     1        6    0.102    0.215    -.3422E+04   -.2146E-03   0.3091E+03
     1        7    0.120    0.201    -.1855E+04   -.2014E-02   0.3317E+03
     1        8    0.137    0.185    -.1380E+03   -.3547E-02   0.3319E+03
     1        9    0.151    0.167    0.1630E+04   -.4515E-02   0.3095E+03
     1       10    0.163    0.148    0.2002E+04   -.6198E-02   0.2530E+03
     1       11    0.172    0.127    0.2841E+04   -.7092E-02   0.2235E+03
     1       12    0.178    0.107    0.3552E+04   -.6921E-02   0.1816E+03
     1       13    0.182    0.086    0.4095E+04   -.5661E-02   0.1340E+03
     1       14    0.183    0.065    0.4461E+04   -.3454E-02   0.8795E+02
     1       15    0.181    0.045    0.4572E+04   -.3443E-02   0.1065E+03
     1       16    0.177    0.026    0.4909E+04   0.2985E-03   0.6738E+02
     1       17    0.172    0.008    0.5220E+04   0.3879E-02   0.4397E+02
     1       18    0.169    0.003    0.5407E+04   0.5019E-02   0.2622E+02
 LAYER       NO.     X1       X2         TXX          TYY          TXY
     2        1    0.004    0.245    -.1400E+04   -.4691E-02   -.3086E+03
     2        2    0.012    0.245    -.1454E+04   -.4289E-02   -.3073E+03
     2        3    0.036    0.242    -.2437E+04   0.1522E-03   -.2835E+03
     2        4    0.059    0.235    -.3237E+04   0.4662E-02   -.2371E+03
     2        5    0.081    0.226    -.3777E+04   0.8923E-02   -.1719E+03
     2        6    0.102    0.215    -.5237E+04   0.6581E-02   -.1695E+03
     2        7    0.120    0.201    -.5151E+04   0.1032E-01   -.7824E+02
     2        8    0.137    0.185    -.4696E+04   0.1352E-01   0.1860E+02
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2	9	0.151	0.167	-.3876E+04	0.1610E-01	0.1139E+03
2	10	0.163	0.148	-.3168E+04	0.1316E-01	0.1447E+03
2	11	0.172	0.127	-.2575E+04	0.1318E-01	0.1930E+03
2	12	0.178	0.107	-.1765E+04	0.1298E-01	0.2273E+03
2	13	0.182	0.086	-.8389E+03	0.1281E-01	0.2454E+03
2	14	0.183	0.065	0.9469E+02	0.1289E-01	0.2478E+03
2	15	0.181	0.045	-.1070E+03	0.1407E-01	0.2533E+03
2	16	0.177	0.026	0.8699E+03	0.1542E-01	0.2432E+03
2	17	0.172	0.008	0.1618E+04	0.1736E-01	0.2330E+03
2	18	0.169	0.003	0.2021E+04	0.1770E-01	0.2342E+03
LAYER	NO.	X1	X2	TXX	TYY	TXY
3	1	0.004	0.245	-.4572E+03	-.8219E-02	0.3086E+03
3	2	0.012	0.245	-.3658E+03	-.8361E-02	0.3073E+03
3	3	0.036	0.242	0.7596E+03	-.1181E-01	0.2835E+03
3	4	0.059	0.235	0.1843E+04	-.1436E-01	0.2371E+03
3	5	0.081	0.226	0.2802E+04	-.1570E-01	0.1719E+03
3	6	0.102	0.215	0.2802E+04	-.2351E-01	0.1695E+03
3	7	0.120	0.201	0.3476E+04	-.2197E-01	0.7824E+02
3	8	0.137	0.185	0.3936E+04	-.1880E-01	-.1860E+02
3	9	0.151	0.167	0.4173E+04	-.1403E-01	-.1139E+03
3	10	0.163	0.148	0.3411E+04	-.1147E-01	-.1447E+03
3	11	0.172	0.127	0.3237E+04	-.8575E-02	-.1930E+03
3	12	0.178	0.107	0.2957E+04	-.4693E-02	-.2273E+03
3	13	0.182	0.086	0.2647E+04	-.2402E-03	-.2454E+03
3	14	0.183	0.065	0.2382E+04	0.4328E-02	-.2478E+03
3	15	0.181	0.045	0.2663E+04	0.3700E-02	-.2533E+03
3	16	0.177	0.026	0.2622E+04	0.8859E-02	-.2432E+03
3	17	0.172	0.008	0.2761E+04	0.1308E-01	-.2330E+03
3	18	0.169	0.003	0.2703E+04	0.1514E-01	-.2342E+03
LAYER	NO.	X1	X2	TXX	TYY	TXY
4	1	0.004	0.245	0.3084E+04	-.2148E-01	-.3624E+02
4	2	0.012	0.245	0.3086E+04	-.2128E-01	-.4183E+02
4	3	0.036	0.242	0.2848E+04	-.1963E-01	-.1229E+03
4	4	0.059	0.235	0.2386E+04	-.1639E-01	-.1953E+03
4	5	0.081	0.226	0.1747E+04	-.1176E-01	-.2530E+03
4	6	0.102	0.215	0.9863E+03	-.1671E-01	-.3091E+03
4	7	0.120	0.201	0.1799E+03	-.9631E-02	-.3317E+03
4	8	0.137	0.185	-.6218E+03	-.1735E-02	-.3319E+03
4	9	0.151	0.167	-.1333E+04	0.6576E-02	-.3095E+03
4	10	0.163	0.148	-.1760E+04	0.7885E-02	-.2530E+03
4	11	0.172	0.127	-.2179E+04	0.1170E-01	-.2235E+03
4	12	0.178	0.107	-.2360E+04	0.1521E-01	-.1816E+03
4	13	0.182	0.086	-.2287E+04	0.1823E-01	-.1340E+03
4	14	0.183	0.065	-.1984E+04	0.2067E-01	-.8795E+02
4	15	0.181	0.045	-.2015E+04	0.2121E-01	-.1065E+03
4	16	0.177	0.026	-.1417E+04	0.2398E-01	-.6738E+02
4	17	0.172	0.008	-.8404E+03	0.2656E-01	-.4397E+02
4	18	0.169	0.003	-.6836E+03	0.2782E-01	-.2622E+02

******************** THE STRESS DISTRIBUTIONS ACROSS THE LIGAMENT OF THE PLATE ***********************

POINT=	1	X1= 0.1271	X2= 0.0026	T11= 0.10461911E+03	T22= 0.43780118E+04	T12= 0.10341868E+02
POINT=	2	X1= 0.1397	X2= 0.0029	T11= 0.29470390E+03	T22= 0.33064404E+04	T12=-0.26063698E+02
POINT=	3	X1= 0.1627	X2= 0.0034	T11= 0.34292025E+03	T22= 0.22448220E+04	T12=-0.79359774E+02
POINT=	4	X1= 0.1962	X2= 0.0041	T11= 0.29215934E+03	T22= 0.15173410E+04	T12=-0.11360285E+03
POINT=	5	X1= 0.2401	X2= 0.0050	T11= 0.21404444E+03	T22= 0.10787318E+04	T12=-0.12442130E+03
POINT=	6	X1= 0.2944	X2= 0.0061	T11= 0.14530340E+03	T22= 0.81755422E+03	T12=-0.11835797E+03
POINT=	7	X1= 0.3591	X2= 0.0075	T11= 0.93844418E+02	T22= 0.65169154E+03	T12=-0.10077351E+03
POINT=	8	X1= 0.4401	X2= 0.0091	T11= 0.55440756E+02	T22= 0.53468656E+03	T12=-0.71020255E+02
POINT=	9	X1= 0.5513	X2= 0.0091	T11= 0.25849230E+02	T22= 0.42057408E+03	T12=-0.36908192E+02
POINT=	10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]